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SMALL MOTIVE POWER.

By HENRY S. H. SHAW, Stud. Inst. C.E.

From Selected Papers of the Institution of Civil Engineers.

I.

THE introduction of small motors has been very extensive of late years, and is still increasing. The object of this paper is to consider, in a brief but systematic manner, the present state of the subject in this country, in order that a comparison may be formed of the relative advantages of the various agents now in use. In order to do this at all satisfactorily, an outline, at least, must be given of the modes of applying these agents for the purpose, and also the uses to which the motors themselves are put; so that, after a few preliminary definitions and remarks, the following method has been adopted:

(I.) The working agents are classified, and the causes of loss by the transformation of energy on a small scale are briefly considered.

(II.) The agents are taken in order, and the types of apparatus for applying them are dealt with and classified, and their efficiency examined.

(III.) Finally, the relative advantages of the agents are considered, with reference to the various objects for which they are applied.

It is difficult to define exactly what is meant by Small Motive Power. Whether the size of the motor, the work performed in a given time, or the object for which it is applied be taken for reference, the difficulty of assigning an exact numerical limit exists, such a limit being after all quite arbitrary; for instance, a very small engine, driven at a considerable velocity by high-pressure steam, might develop as much power as an ordinary water wheel, or a much larger engine, under different conditions; but though the term in question would include the first, it would commonly be held to exclude the two last; therefore, it will be interpreted as generally received by engineers, and will be taken to exclude locomotives and marine engines, factory and mill motors, but to embrace the large number of engines for what may be called auxiliary purposes, which are being made more and more to supersede muscular energy.

The only direct sources of power at present used for small motors are fuel and a natural head of water, fuel including every kind of substance which by chemical action, rapid or slow, develops kinetic or actual energy. No source of power can be utilized without some loss of energy, and the "efficiency" of the apparatus used is the exact numerical value of the ratio of the useful work produced by it, to the energy expended upon it, by means of the working substance or agent; this quantity is, therefore, always some fraction less than

unity. Before the final application of energy takes place, it may undergo a number of transformations by means of various agents, two or more being often employed in the case of small motors—as, for instance, when steam power is used to produce an artificial head of water or a current of electricity, which latter may produce work; but since the resultant efficiency is the product of the fractions representing that of each intermediate piece of apparatus, the final advantage to be gained must be balanced against the loss necessarily entailed by the use of each extra agent.

The same source of power may be applied to impart energy to motors by various agents and in different ways, and in order to avoid confusion, in stating the efficiency of an agent, the source of power should also be stated; thus, taking gas as the source of power in the gas engine, and coal in the steam engine, the result of comparing these agents is very different to that obtained by taking coal as the source in both cases, and introducing as a factor in the former case the efficiency of the gas-making apparatus.

The extent to which an agent is to be used may have much to do with determining its suitability for any purpose, and may considerably modify its application; indeed, the choice of the agent may depend on very different considerations, according as it is to be used on a large or a small scale. The importance of this will be shown hereafter, and leads to the agents applicable for use with small motors being more numerous than with large ones.

Section I.

(1.) The agents may be thus classified:

Steam	. . . . .	Heat engines.
Gas	Coal . . .	
	Petroleum, &c,	
Air	Heated . .	
	Compressed	(*a*)
Water	Natural head	
	Artificial head	(*a*)
Electricity	By direct chemical action	
	By dynamo-electric machine .	(*a*)

The agents classed under "heat engines" form the most important portion, and will be, to a certain extent, treated of together, a heat engine being regarded simply as an apparatus by which the action of heat is caused to expand the agent, and thus to do work. Those marked (*a*) (*a*) (*a*) may be regarded as merely transmitters of energy, and acting in the secondary way already referred to. The respective efficiencies, of two of these, and also of steam, for this purpose, have been dealt with in a paper by Mr. Robinson,* M. Inst. C. E., from which paper it appears that water stands the highest, but the loss of head, varying inversely as the fifth power of the diameter of the transmitting pipe would cause its utility to diminish rapidly with the size of the motor. It is only recently that electricity has been much thought of for this purpose, or at any rate has been practically applied, and that agent is treated in Section II. almost entirely with reference to this object.

Before proceeding to examine the efficiencies of the agents separately, it will be well to point out the losses of different kinds attending the transformation of energy, which are common to all. These take place,

(1.) In dissipation, caused by the tendency of the agent to lose the energy imparted to it by communicating it to surrounding bodies, as, for instance, by conduction and radiation in the case of heat, and by leakage with a head of water.

(2.) In the necessary rejection of energy due to the conditions of working.

(3.) In the mechanical arrangements, in connection with which there are two principal losses, the one by friction, which is unavoidable, but may be reduced to a minimum by judicious design, the other by an improper supply of the agent, which supply may be either insufficient or excessive; in the former case the friction of the working parts absorbs an increasing proportion of the energy expended; in the latter the velocity of the motor increases, until the resistances, both useful and prejudicial, are equal to the power applied to overcome them.

As already stated, heat engines are most important among small motors, the use of gas and hot air being almost entirely applied to operations on a small scale; therefore, it seems appropriate to

* *Vide* Minutes of Proceedings Inst. C.E., vol. xlix., p. 1.

deal in this place with the losses common to them.

The first cause of loss, viz., by conduction and radiation, is proportionately greater for small than for large engines, not only from the fact that the heated agent is generally contained and conveyed in cylindrical vessels and pipes, in which the volume varies as the square of, but the surface only directly as, the diameter, and thus the radiation is proportionately greater, but loss by leakage and imperfect lagging is also generally greater; furthermore, the loss of heat in its transference from the fuel to the agent is greater, for, as will be shown by the results of experiments, the conditions of economical combustion are harder to maintain. Both these sources of loss may, however, be considerably modified by mechanical arrangements.

The second cause of loss, from the necessary rejection of heat, is often overlooked. There is not space to examine the question as fully as it deserves, but the conclusion to be drawn is that the agent should be used between as great ranges of temperature as practicable. This is an important point in discussing their relative efficiency, and since the proportionate range for hot air is greater than for steam or gas, the efficiency of the former in this particular is the higher; but at the same time this very quality renders it less capable of developing so much power, since only small differences of pressure accompany this range, which is limited by practical considerations, such as lubrication, packing, &c. There is, however, another and a more obvious way of regarding the action of these agents so as to compare their efficiencies; the heat rejected in the waste steam or gas has to be entirely restored to the new portion of the agent, by the expenditure of fuel, while in the air engine this is partly effected by compression in the return stroke; and so, though the loss of fuel is not so great in the air engine, the high-back pressure resulting from compression makes the resultant work also not so great, and leads to the conclusion, which is verified in practice, that air engines are not so powerful as steam or gas engines, though more economical.

The higher the initial temperature of the agent the greater will be its possible efficiency, for the lower limit cannot at the best differ much from the temperature of surrounding bodies, but the upper is only limited by mechanical difficulties; and though it may be shown that it is impossible to abstract all the imparted heat in the form of work, the proportion of heat so abstracted being, in fact, never greater than one-seventh in the best steam engines of the day, yet the efforts made in this direction must be attended with success in proportion as those difficulties are overcome. In the discussion which followed the reading of Mr. Flannery's paper* on high-pressure steam boilers these difficulties seem to have formed the only ground of objection to their use. Pressures of from 300 to 500 pounds per square inch have been successfully used by Mr. Perkins, and the application of high pressures and speeds is greatly increasing for small steam engines.

The third cause of loss, viz., that from friction and imperfection of machinery, has a special interest from the high speeds now used in some small engines. It may be regarded as certain that the proportionate loss from these causes is greater on a small than on a large scale; but, again, the proportionate waste of fuel is nearly always of less consequence. From the recently issued first reports on "Friction at High Speeds," of the Institution of Mechanical Engineers, it would appear that, at any rate up to the velocities at present used, the friction seems slightly to decrease as the velocity increases.

Section II.—steam.

Steam engines are at present by far the most extensively used of small motors, some on a very small scale being made to develop considerable power.

The generation of steam on a small scale is almost entirely limited to vertical boilers—at any rate, for purposes on land—though the horizontal form, as well as sectional boilers, are also coming into more general use, the portability of the first being a strong recommendation. Figs. 1, 2 and 3 are sections of vertical boilers in common use, the general form being a cylindrical wrought-iron shell containing a furnace, also cylindrical

* *Vide* Minutes of Proceedings Inst. C.E. vol., liv. p. 123.

and concentric with it, and fired through an opening in one side. In other particulars there is considerable difference. In Fig. 1, which, though extensively used, is deservedly falling into disfavor, the gases pass upward to the funnel through a number of vertical tubes, either a cast-iron top or an enlarged crown being necessary to allow their escape to the funnel; this latter device is shown in the figure, and also an ingenious arrangement for preventing the earlier destruction of the central tubes, by the tendency of the flames to ascend

Fig. 1.

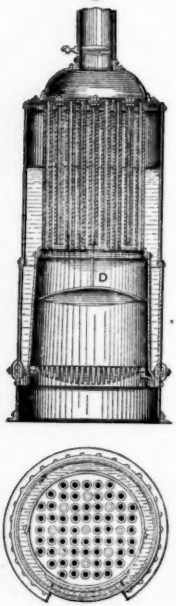
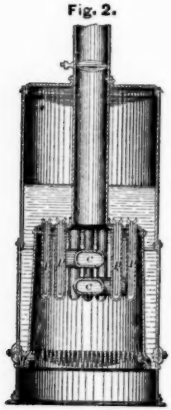


Fig. 2.



through them, which is especially the case when the fire is low. This consists of a concave wrought-iron plate (D), by whose under side the gases are reflected and are compelled to ascend round its edge, where combustion of some portion of them takes place, and the side of the furnace is acted on by the flame instead of by the current of cold air, which otherwise would be drawn up and be in contact with it. Fig. 2 is a still more common form, the gases passing up a central flue; but there are a considerable number of different devices for robbing them of their heat before this takes place; the oldest and most usual is that of the cross tubes *c*, *c*, whose section is, however, generally circular; these have the advantage of adding to the strength of the furnace; the furnace, besides being subjected to external pressure, is the vulnerable part in the, by no means uncommon, event of shortness of water. Another plan is to have pendent tubes *p*, *p*, *p*, of which the "Field" type is a good example, as it allows a complete circulation of water by separating the ascending and descending columns, by an inserted tube shown in the diagram. The success of this plan is shown from the fact that no less than 134,700 are now in use. Another good device is applied in the "Davy-Paxman" boiler (Fig. 4), in which the tubes, instead of ending in the furnace, pass down and communicate with the water in the lower part of the boiler, and so establish complete circulation, which is prevented from acting too violently by bafflers placed at their upper ends. These and various other

arrangements, though adding to the complication of the boiler, are very desirable if effective in action, for the loss of heat in them, which experiments indicate, is thereby much diminished. Thus the boiler of Latham and Bradley, 10 feet high and 4 feet in diameter, has 132 square feet of heating surface. Fig. 3 is an excellent design known as the "Talbot," which presents considerable heating surface; the furnace is of cast iron, which in a recent modification is lined with firebrick, and the ash box acts as a water heater. These boilers have given good results in practice, and the advantage of a non-tubulous boiler with large heating surface is realized by those who have experienced the cost and trouble which tubes often entail. Fig. 5 is a view of an ordinary sectional boiler, and it will be seen that the amount of heating surface exposed must cause it to be a rapid steam generator. The arrangement *a*, *a*, *a* of firebrick and the iron plates *b*, *b*, *b* divert the flames, and compel them to pass well round the tubes, after which they escape by the flue *c*. A portion of the steam space is shown at *d*. Another form of sectional boiler may be briefly described as a square box of tubes, which run from side to side in two directions, and through which the flames are made to pass successfully. Yet another form, which is used in America, consists of a series of cast-iron spheres with necks, which are faced, and so connection is made by means of long bolts. The advantages of sectional boilers are summarized in the paper of Mr. Flannery,* where there is also a quotation from the third report of the Admiralty on the subject. There is a large number of inventions of both vertical and sectional boilers, but the foregoing examples are typical of the forms generally used. A small vertical boiler commonly has the engine attached to it, and forms its support; this may be effected by bolting the engine either to its side, or by means of a cast-iron bedplate or a wrought-iron ring securing it upon its top. Portability is thus secured, and the evils arising from alternate expansion and contraction of the boiler,

Fig. 3.

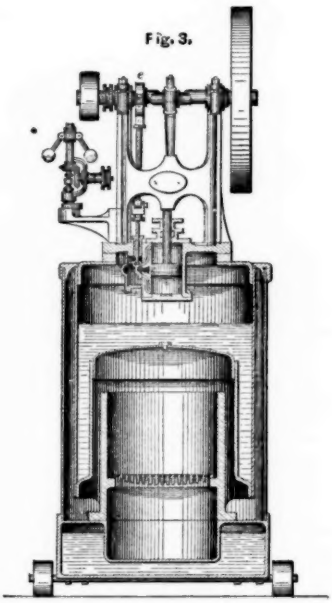
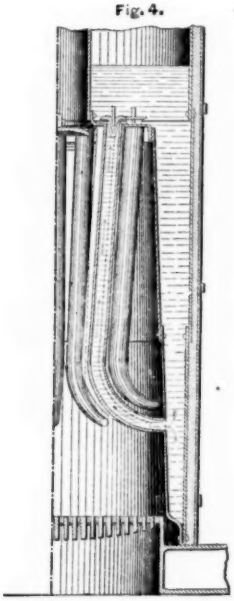


Fig. 4.

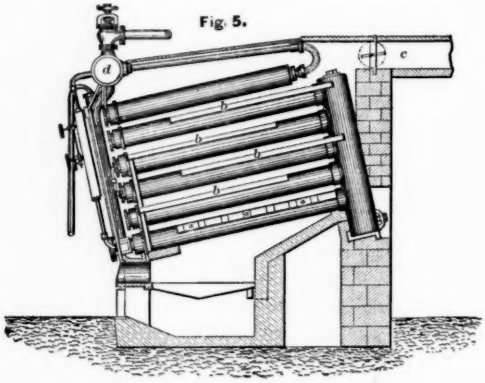


* *Vide* Minutes of Proceedings Inst. C.E., vol. liv. p. 128.

which would be felt in a large engine, are not in this case appreciable, chiefly because when used on a small scale the engine is only connected with a fixed external object by a belt or bearing at a little distance from it; but it has been remarked that with small marine engines, where the case is different, the objection is considerable. One excellent example of the latter method is afforded by the design shown in Fig. 3, in which the cylinder is let into the boiler, and, besides the benefit of short steam connections, a perfect steam jacket is formed.

The efficiency of small boilers is a most important thing to consider, and, so largely are they at present in use in

Fig. 5.



in conjunction with small engines, that it would be impossible to form a satisfactory comparison between small motors without some reliable data on the subject. Now the paper of Mr. Flannery indicates a high efficiency for sectional boilers, but these are not used to anything like the same extent as the vertical form; and on turning to different sources for information, the author was unable to obtain independent results. For instance, in the comprehensive manual of Mr. D. K. Clark,† M. Inst. C.E., although boilers of marine, locomotive, stationary, and portable engines are fully treated, yet the class in question is not mentioned; and the ratio of heating surface to grate area and other conditions

† A manual of "Rules, Tables, and Data for Mechanical Engineers." Blackie and Son.

are so different in these latter, that no comparison can fairly be drawn. This information being required, and also data concerning the working of small steam engines of the ordinary type, through the kindness of Mr. Fry, Managing Director of the Bristol Wagon Works, and of Mr. Arrowsmith, Steam Printing Works, Bristol, two series of experiments were made upon a 2-HP. vertical engine and boiler, manufactured by Dodman, of King's Lynn, belonging to the former, and a 4-HP. horizontal engine called the Soho, and a vertical boiler made by Tangye, of Birmingham, belonging to the latter. The data now given will refer only to the boilers, that of the engines being introduced hereafter. During the two days' trial of the former, which will be called No. 1, and one day's trial of the latter, No. 2, the difficulty was experienced of keeping uniform the quantity of water and the pressure of steam; but during a run of three hours on No. 1, and of four hours with No. 2, by careful stoking and attention, and by taking observations every quarter of an hour—of revolutions, pressure of steam, and supply of feeding water—a close approximation to regularity was obtained. Both the dynamometer and the indicator were used, and the quantity of feed water and coals carefully weighed, and, as will be shown in No. 2, the volumes of steam used agreed fairly well with the volume of the cylinder and ports obtained by actu-

ally filling these spaces with water afterwards, so that little, if any, priming could have taken place in No. 2, while in No. 1 the difference must not be overlooked, and the boiler efficiency in this case can only be approximate. No. 1 was of the form shown in Fig. 1, and was well lagged all over; No. 2 was of the form represented in Fig. 2, but was deficient in heating surface, having no pendent tubes and but one cross tube, circular in section.

Table of Results of Experiments on Small Vertical Boilers.

Boiler.	Height.	External diameter.	Number of tubes.	Diameter of tubes.	Length of tube.	Diameter of furnace.	Area of grate in sq. ft.	Height of furnace in feet.	Heating surface in sq. feet.	Ratio of heating surface to grate area.	Mean pressure on boiler in lbs. per square inch.	Water evaporated per hour in pounds.	Coal used in lbs.	Evaporation in lbs. of water per lb. of coal from 100° F.	Efficiency of boiler.
	Ft. in.	Ft. in.		in.	Ft. in.			Ft. in.							
No. 1.	4 6	2 6	7	2½	2 2	..	1¾	1 9½	21.2	12.0	51.5	121	36⅝	3.3	0.26
No. 2.	5 4	2 6	1	8	2 0	..	3½	3 0	26.8	8.54	32.0	170	34½	4.9	0.39

The feed water was warmed to about 100° Fahrenheit. The coal used with No. 1 was Parkfield; an analysis, with which the author was kindly furnished by Mr. Handal Cossham, shows 82 per cent. of carbon, 5.6 per cent. of hydrogen, and 6.1 per cent. of oxygen, and has a heating power of 14.3 pounds of water, at 212° Fahrenheit, converted into steam; so that, theoretically, the evaporation should have been from 100° Fahrenheit, about 12.5, instead of 3.3 ∴

$$\text{Efficiency} = \frac{3.3}{12.5} = 0.26.$$

With No. 2 the evaporation should have been about the same, making its efficiency = 0.39. Thus, although the ratio of heating surface of No. 2 was considerably less than No. 1, the evaporative power was greater, which seems only to be accounted for by the fact that most of the heating surface of the latter was vertical. If the average evaporative power of large boilers from 100° Farhenheit for this kind of coal be taken at 0.65, it is seen how much inferior these small boilers are to larger ones, although from what has been said it is seen that these boilers are being superseded by more efficient ones, with larger heating surface, as Latham's and Bradley's, which, though little larger than No. 2, has five times as much heating surface, and the field, of which there are 1,380 in use; but the introduction of more heating surface by tubes, &c., offers a larger surface for corrosion and leakage, and therefore involves more repairs; and the general conclusion is that the necessity for small boilers is the weak point in the use of steam on a small scale.

There are four ways of using steam on a small scale, which seem to lead to the following classification:

(1.) Single-acting engines using high pressures and speeds.

(2.) Double-acting engines using moderate pressures and speeds.

(3.) Rotary engines.

(4.) Apparatus for causing the pressure or impulse of steam on fluid surfaces to directly raise or force that fluid.

The first three are always non-condensing engines; the last kind can hardly be called engines, being apparatus of the injector or pulsometer type; but as they are used principally as a substitute for other kinds of steam pumps, or manual operations, they must certainly be considered as included by the term Small Motive Power. Thus it may be said that all small steam engines are non-condensing. This is obviously because the gain due to the partial absence of back pressure, which condensation secures, would not be sufficient to compensate for the increased complication and

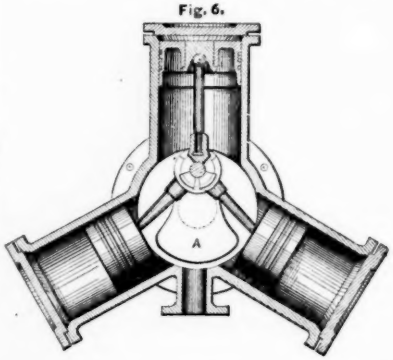
first cost which would at the same time be entailed; while the difficulty of maintaining a good vacuum increases, as also does the proportionate work done by the air pump, as the size of the engine diminishes. This fact has an important bearing on the pressure used; for it must be remembered that the effective pressure is the difference between the forward and the back pressure, the latter being, in the above cases, nearly constant. Low forward pressures must be very wasteful, the limiting case being when the forward and back pressures are equal, for then the whole imparted heat would, in a non-condensing engine, be absolutely thrown away.

SINGLE-ACTING ENGINE.

(1.) These engines have been largely introduced for high speeds, so as to save the necessity of intermediate gearing, and to develop considerable power with a small motor. They almost invariably have multiple cylinders, generally three, as in the well-known Brotherhood example, and in those of Watt, Wigrell and Hasley, and Willans; while Vosper's engine has four. The steam only acts on one side of the piston. The great advantage of this is, that there is no shock from the wearing of the connecting-rod ends, or "backlash," as it is called. This is most important, at the almost incredible velocities at which some of these little motors work—many by the first-mentioned maker working at 1,400, and some even at 1,600 to 1,800 revolutions per minute. Moreover, the piston rod and guide blocks are dispensed with, and parallel motion is secured by having a deep piston, to which the connecting rod is attached, and thus compactness is obtained; the absence also of eccentrics and eccentric rods renders the working parts few in number, while the use of the flywheel is entirely obviated. Of the numerous forms, that of Brotherhood has by far the most extensive application; and it may be said that the one exhibited by the inventor at the Vienna Exhibition, in 1873, which gained a silver medal, was the first of the class of which several hundreds have been made by him alone, and a large number by other inventors, who have since worked in this direction. It is thus seen that there is a movement in favor of a system by which considerable power is obtained from a small motor. Of this type a brief description of the Brotherhood engine only will be given, as engravings and descriptions of the others are to be found in comparatively recent numbers of various engineering journals.

Fig. 6 is a section through the three cylinders of a Brotherhood engine; the crank being balanced by a counter weight A, and the three connecting-rod ends merely butting against the crank pin, to which they are held by two steel rings. The valve is a rotary one, and its action is shown in Fig. 7, which is a longitudinal section. The steam is being ad-

Fig. 6.



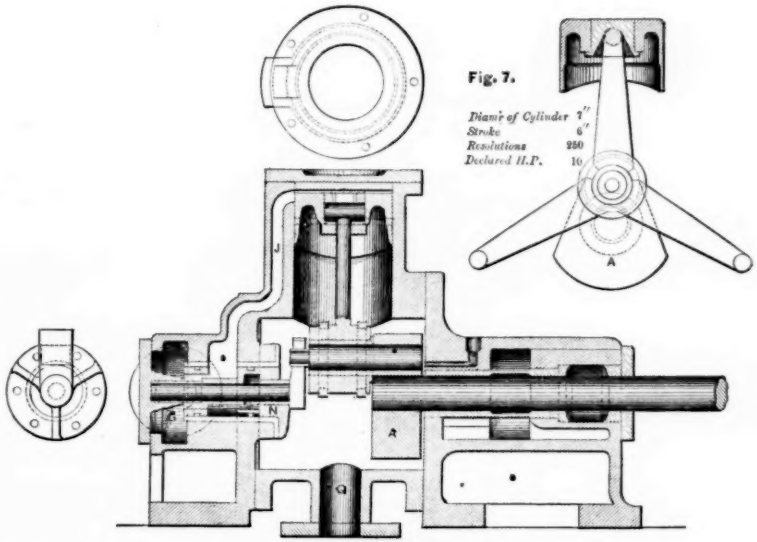
mitted to the top of the piston by the port J, the opening in the valve putting it in communication with the steam chamber G. The exhaust is made by the passage N′ being placed in communication with the annular space in which the crank works, and finally escapes through the exhaust pipe, Q, placed at the bottom of this chamber. The steam pressure was about 70 lbs. per square inch, with 250 revolutions per minute. The compression, which commences rather before half-stroke, can scarcely be a source of loss, as it allows the entering steam to rise to the full pressure at once, and certainly results "in the complete suppression of all species of shock." This, in a single-acting engine, with no arrangement for tightening the connecting-rod bearings, is a desideratum. The wire-drawing, obvious from the diagram, is the result of the circular valve, which, advantageous as it is for many reasons,

is not moving at its greatest speed as with the ordinary slide valve, when the engine is near its dead centers. Fig. 8 shows the period of its action. The efficiency of the machine itself, that is the ratio of the indicated work to that obtained at the brake, is 75 per cent. The developed power was 10 HP. It is to be regretted that no determinations of fuel and steam used were taken. But although it has been asserted that these engines are wasteful of steam—and certainly the length of steam passage must entail a loss—yet on some trials made by the Royal Engineers, in which these engines were used to drive Gramme machines, the consumption of steam was 41.1 lbs. per hour per HP.,* revolutions 500 per minute, pressure of steam 80 lbs. per square inch, about 13 HP. being developed. This agrees with the consumption of steam obtained by officers of the French navy, which, the inventor states, was 40 lbs., and is equivalent to an expenditure of from 4.5 to 5 lbs. of coal with a good boiler, or 8 lbs. with one of the type already examined.

* "Engineering," vol. xxiii., p. 320.

DOUBLE-ACTING STEAM ENGINE.

(2.) The second class of small engines, viz., those which are double acting, and generally work with a single short D-slide-valve, are at present by far the most common, and a section of one of this type is shown (Fig. 3). Now, although the forming is made to take every concievable shape, and they are made to work in every position, and have slight differences of detail, yet the principle of action is the same, and is so well known as to need no explanation. The



BROTHERHOOD'S 3 CYLINDER ENGINE

point of cut-off of the steam is from $\frac{5}{8}$ to $\frac{3}{4}$ of the stroke, the corresponding grade of expansion being from $1\frac{3}{5}$ to $1\frac{1}{3}$, and more expansion than this cannot be economically effected with a single valve. The two engines before referred to cut off steam at about the latter point. The details of the experiment were as follow: Both the indicator and the dynamometer were used, the latter being of the ordinary form, that is Prony's dynamometer lined with blocks of elm, embracing pullies respectively 2 feet 7 inches and 3 feet in diameter in the two cases. No. 1 was a vertical engine, and

was bolted to the boiler. No. 2 was horizontal, and rested on a foundation of brickwork and stone. The number of revolutions varied to some extent, but were taken every quarter of an hour.

Fig. 8.

The experiments would perhaps have been more satisfactory if a speed counter had been applied, since with a brake on a small engine the action cannot be kept perfectly regular, and the governors were purposely disconnected. However, the following Table is without doubt a near approximation to the truth:

that the final efficiency of the whole is only about 0.19 in No. 1, and 0.3 in No. 2, that is theoretically possible for an engine using steam at those pressures and with those grades of expansion, but employing a condenser and having a boiler of 0.7 efficiency.

An ingenious kind of steam engine is used in the engineering laboratory, University College, London, invented by Mr. H. Davey, M. Inst. C.E., of Leeds, which is the only one of its class yet made. A small quantity of water is injected at each stroke into a coil of pipe heated by a furnace, and is at once converted into steam, and so drives the engine, which is single-acting, about ¾ HP. being developed. Improvements are being made in the design, and Professor Kennedy, M. Inst. C.E., thinks the invention a promising one.

Small oscillating steam engines have been used, but the admission and release

TABLE OF DATA FROM ENGINE TRIALS.

Engine.	Diameter of cylinder.	Stroke.	Number of revolutions per minute.	Average pressure in boiler, by gauge, in lbs. per sq. in.	Mean pressure in cylinder by indicator, in lbs. per sq. in.	Indicated HP.	Developed HP. (brake).	Efficiency of machinery = D.H.P. / I.H.P.	Steam in lbs. per hour per I.H.P. by weighing feed-water.	Steam in lbs. by calculation.	Coal in lbs. per hour per HP.	Efficiency of engine supposing efficiency of boiler=0.7.	Real final efficiency of boiler and engine.
	Ins	Ins											
No. (1) 2 HP. nominal.	4¼	7	130	51.5	39.0	2.57	2.011	0.78	47.2	42.32	14.3	0.035	0.013
No. (2) 4 HP. nominal.	6½	7½	133	32.0	22.3	3.75	3.040	0.80	45.3	44.0	9.2	0.039	0.022

These results show that in this class of engine, at any rate, the consumption of fuel is at least from four to seven times greater than in the best kind of large engines, and that it is therefore beside the question to quote the performance of large engines when comparing the working cost of small motors. In connection with the indicated power of No. 2, which is less than the nominal, it is only fair to state that the engine often works with about 40 lbs. pressure and 180 revolutions, which would cause it to indicate more than 6 HP. A similar remark applies to No. 1. Again, it is well to note

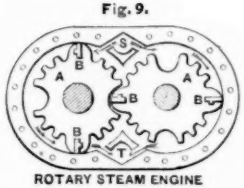
of steam, to be effected economically, require rather complicated valve gearing.

There is another class of double-acting steam engines which are not rotative, and since merely reciprocating action is obtained, the slide-valve has to be driven by some arrangement other than an eccentric. Such engines are often used as donkey pumps and often as steam rock-borers. An instance of the former is the "Special Steam Pump;" a description of its action is to be found in the pamphlets of Thurston or Evers on the steam engine.

ROTARY ENGINES.

(3.) It was stated in "Engineering," six years ago, that "most mechanical engineers now-a-days have a deeply-rooted antipathy to rotary engines." Nor has the appearance of any successful patent between that time and this caused an alteration in opinion. Rotary engines are understood to be those which obtain circular motion directly by the action of steam upon some revolving piece, and so avoid the necessity for the conversion of reciprocating into circular motion. It is not to be wondered at that a large number of ingenious attempts should have been made to thus obviate the use of the crank and the connecting rod For instance, thirty years ago, one of the form known as a "disc" engine, was for some years years used to drive the printing machinery in "The Times" office, and is described by Rankine;* but although according to that author "the number of rotatory engines which have been patented in Britain alone is certainly upwards of two hundred," yet "very few have been brought into practical operation, and those to a limited extent only; for their friction and liability to wear have been found to be greater than those of ordinary engines, and they have no advantage except compactness." There is, however, one purpose to which they have recently been applied with some success, viz., to fire-engines, where economy of fuel, which the difficulty of keeping the revolving portion steam-tight renders impossible, is not the chief point, and where the use is only occasional, and so the wear not of the first importance. Fig. 9 shows a simple

Fig. 9.



ROTARY STEAM ENGINE

form which is used for this purpose. Here the steam entering at S drives round the discs A, A, in opposite directions, and, finally escaping at T, A, A, give motion to two similar discs, which act as the pump by a similar action, and so great compactness is obtained. The method of packing is ingenious and effective, and is formed by the metal pieces B, B, being pressed outwardly by springs, their centrifugal force acting with a similar effect. Behrens' engine is similar in action, and is much used in America as a pump.

* "A Manual of the Steam Engine and other prime movers," p. 504.

STEAM ACTION ON FLUID SURFACES.

(4.) Two kinds of action are used. The first, Mr. Bramwell, F.R.S., M. Inst. C.E., has said, is expressed by one word "Concentration," and may be said to act by impulse, to which belong the Giffard Injector and the Inspirator. The second uses the principle of the Savery engine and acts chiefly by pressure, of which the Pulsometer and Pulsator are types. As before stated, this class of apparatus is employed for pumping purposes, and this may be said to be its exclusive duty; its use in this direction is increasing, and whenever there is a supply of steam the simplicity of action and remarkably small weight and size of the whole machine are great recommendations.

The theory of the action of injectors is now well known, so that it will not be necessary to describe any one of the numerous forms in use, although a reference to Hancock's inspirator may be made. This little instrument is becoming very popular, as it admits of accurate adjustment, after which little attention is required; while the weight of one capable of feeding a 48-HP. boiler is only 6 lbs. The undoubted function of all injectors is the foregoing, viz., the feeding of boilers, and they are universally applied to locomotives for this purpose.

The second kind are really steam vacuum pumps. The "Pulsometer" is a very successful example. The action is automatic and alternative, and the arrangement is, in fact, a self-acting Savery engine. As with the Savery engine, however, the consumption of fuel is considerable. Trials were made at the Cincinnati Exhibition in 1875, quoted by Mr. D. K. Clark,* and at the Friedrich Lead Mine, Upper Silesia, recorded in the

* "A Manual of Rules, Tables," &c., p. 969.

Abstracts of Foreign Papers.* In the former case, 53.1 lbs. and 43.4 lbs. of coal per hour per HP. for water actually raised were expended on two respective trials; in the latter 52.4 lbs. of slack were used for the same amount of work; but in the latter trial a "Tangye" pump consumed 40.5 lbs. for the same duty. The conclusions drawn from this trial, and other data obtained by experience, were that the total outlay for stores and repairs with pulsometer was 1.68d. per hour per HP., that for the piston pump being 1.44d. The great recommendation of the former, however, is its portability, as it is as well able to work when slung by a chain as in any other way; also it will, in the words of the advertisement, "pump almost anything," including tar, sewage, chemical liquids, and even liquid cement, and is one-half cheaper in first cost than an ordinary pump of equal power; thus there is evident reason for the extensive sale which it enjoys. A recent modification, under the name of the "Hydrotrophe," is being employed to feed boilers, and both this and the injector must be economical, the surplus heat imparted to the water being returned to the boiler.

GAS.

Gas is a convenient source of power; and in towns, where it is already supplied almost to every building for purposes of illumination, it is at once obtainable in any moderate quantity. Moreover, the principle of adopting a large centre of supply, which is employed in the case of gas, is, on scientific grounds, truly economical, for that substance is supplied in the state of fuel, and involves no more loss in its transmission than would occur in the transference of any other fuel. But in spite of many attempts to use it for small motors, it has only recently been much adopted. Air is used in a peculiar way in the gas engine, and it is only in a similar way that gas can be profitably employed, viz., as the working agent, at the same time as the source of power; for the calorific power of gas, taken in conjunction with its present cost, at once puts out of the question any idea of using it as a direct substitute for coal.

The original method of using gas in this way was to admit it to the cylinder of the engine, together with air in such proportions as to form an explosive mixture, which was ignited, and work obtained by the consequent expansion. Lenoir's engine, which was one of the first, resembled an ordinary horizontal engine, effecting the ignition by an electric spark; but the difficulty of utilizing this sudden effect was a cause of considerable loss, and it is stated by a German writer on the subject* that 3 cubic metres (about 106 cubic feet) of ordinary coal gas are required per hour per HP. for a small sized engine. This corresponds to the figures quoted by Mr. D. K. Clark† from the experiments of M. Tresca, two successive trials of this engine, giving 112 and 97 cubic feet respectively, the mean of which is 104½ cubic feet per hour per HP. at the brake, being an efficiency of little more than 4 per cent. Though this engine had an extensive sale in Europe, it was to a great extent superseded by the Otto and Langen motor. In this the sudden expansion of the gas was allowed to raise a piston against the pressure of the atmosphere alone, which pressure, upon the cooling of the heated gas in the cylinder hastened by a water jacket, drove down the piston and so performed work, the piston-rod being a rack which geared with a pinion on the horizontal shaft. This engine is a regular and efficient motor, and its consumption of from 22 to 30 cubic feet of gas per hour per HP. is vastly superior to the Lenoir engine, and is even less than in some engines which have superseded it. One engine, which has been at work for ten years in Bristol is in good condition, and has cost during that period very little for repairs; but its noise in working is much against its general adoption. Again, it is necessarily a vertical engine, by no means so popular as the horizontal form, and is anything but compact. Thus when the new engine of Otto was introduced into this country by the same manufacturers, Messrs. Crossley Brothers, of Manchester, which was at once compact, economical, and above all, silent, they almost at once ceased to make the Otto and

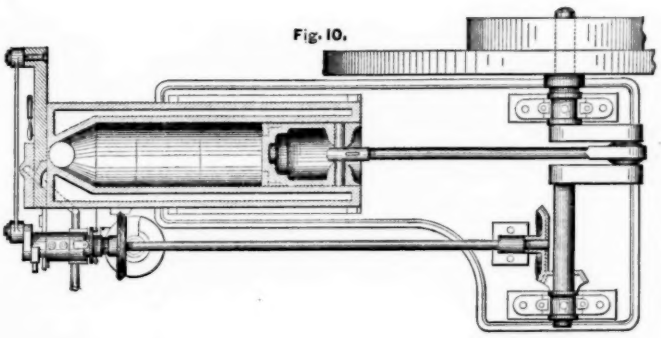
* *Vide* Minutes of Proceedings Inst, C.E., vol. lvi., page 371.

*"Klein-Kraft-Maschinen," p. 27. Von Peter Hell, Braunschweig.

†"A Manual of Rules, Tables," &c., p. 921.

Langen, and have already sold, since 1877, upwards of eight hundred of what are known as the Otto Silent Eugine, more than two thousand being the number sold in Great Britain and on the Continent. The principle of action employed, as indeed of nearly all the many kinds since patented, is intended to meet the difficulty of utilizing the energy of the gas consequent upon the sudden explosion in former engines. The method of doing this is best stated in the words of the specification of the English Patent, No. 2,081, 1876: "A combustible mixture of gas or vapor and air is introduced into the cylinders, together with air or other gas that may or may not support combustion in such a manner that the particles of the combustible mixture are more or less dispersed in an isolated condition in the air or other gas, so that on ignition, instead of an explosion ensuing, the flame will be communicated gradually from one combustible particle to another, thereby effecting a gradual development of heat and a corresponding gradual expansion of the gases, which will enable the motive power so produced to be utilized in the most effective manner." Fig. 10, is a sectional plan taken from the same source, and is similar to the engine now manufactured, except that a single instead of a double crank is used. "The cylinder is constructed of greater length than the stroke of the piston, so that there is a space beyond the latter when it is at end stroke. Assuming this space to be filled with a portion of the gaseous products of combustion resulting from the last stroke, the piston, in performing one part of its stroke, draws in atmospheric air, after which it will draw in combustible mixture during the remainder of the stroke." These three strata of gases are compressed in the return stroke, and ignited, when the principle above quoted comes into operation; and the next forward stroke does useful work, the fourth, or next return stroke, expelling the product of combustion. The slide valve is made alternative in its action, by being driven through a bevel wheel on the main shaft of half the diameter of that on the slide-valve rod. The mixture used is 1 part of gas to 16 parts neutral; the latter being made of air and products of combustion in the proportion of 5 to 3. Some recent inventors adopt the main principle involved, but they employ a separate compression cylinder, and by advertising a combustion at each revolution, seem to imply the alternate action to be objectionable; and it must be confessed that the Otto engine is not perfectly regular in its action with a light load, combustion taking place perhaps only once in five or six revolutions. At the same time, with a proper load and a moderately heavy flywheel, this is hardly appreciable, and is of undoubted benefit to the cylinder, which, being also single-acting, is thereby only heated once in four strokes. One such engine has been running day and night at the Hinckley Gas Works for fourteen months, this being equivalent to three years' ordinary work, without needing any repairs, which, notwithstanding the water-jacket surrounding

Fig. 10.



these cylinders, is a satisfactory results and is contrary to many predictions concerning it. Such a result may possibly not attend those burning gas at every revolution.

The latest trials of the amount of gas used are, according to Mr. Crossley, 24 cubic feet per hour per HP. for the 1 or ½ HP. engines, and 18 cubic feet for the 16 HP. engines.

Another engine, the Eclipse, of Messrs. Simon Beechey & Co., of Nottingham, with a separate compression cylinder, has a novel feature, in the use of steam, which is generated by the heat abstracted from the cylinder in order to keep it moderately cool. This steam is used in the cylinder not only to assist the action of the expanding gas—which it certainly does, being capable of driving the engine for several minutes after the supply of gas is turned off,—but it also acts as a lubricant, by which it is claimed that a great saving in oil and tallow is effected. Its action is fully described, and drawings are given in the "Engineer," 1879,* and the "Textile Manufacturer," May, 1879. In the former it is stated that in a trial of one, the working cylinder, which was 8 inches in diameter, with 16 inches stroke, indicated 6.03 HP., and the compressing cylinder, which was 7 inches in diameter, with 10 inches stroke, indicated 1.87, leaving 4.16 HP. as the net indicated HP. of the engine. The consumption of gas was from 22 to 26 cubic feet per HP. per hour.

The objection to the foregoing type of engines is that they have to be started by hand and turned through a revolution or two; although the makers of the latter engine are said to have obviated the necessity for this by an invention. With very small powers this objection is not felt, nor is it with a recent novelty, the Bisschop. This little motor, which is made from 1-man power upward, has had a sale of three hundred during the first nine months since its introduction: the consumption of gas is greater in proportion than in other larger gas engines, but its first cost is very small, and being advertised to work for ½*d.* per hour, although by no means a silent engine, it has decidedly met a long-felt want.

Vide vol. xlvii., p. 3.

The theory of gas engines is yet imperfect; but some things are certain with regard to them, and one is, that the absence of a boiler in connection with them gives a great advantage over the steam engine. The absence of risk either from explosion or conflagration is another strong point; while the scientific application of production on a large scale, in the form of the gas used, is perhaps the strongest of all, in leading to the conclusion that their present popularity is likely to last, gas engines being a step in the right direction.

HYDRO-CARBON MOTORS.

Another class of engines must be alluded to, although not much in use in this country, viz., hydro-carbon motors. In America, where petroleum is abundant, successful attempts have been made thus to apply it, and its use is now rapidly extending. The principle of action is much the same as with a gas engine. In the Brayton motor, described in "Engineering,"* a jet of air is forced into fibrous material soaked in petroleum, and this substance is deposited on the meshes of wire gauge, from which it is evaporated. The vapor forms with the supply of air a combustible mixture, which is, as in the Otto silent engine, not explosive, but admits of gradual combustion, with the attendant advantages. In Germany the Hock motor is commonly adopted; its appearance is similar to an ordinary horizontal engine, and the German writer already quoted† describes it as sucking in a small quantity of the fluid to about ⅓ of its stroke, and also a certain quantity of air, when ignition and gradual expansion take place through the remaining ⅔. Brayton's motor, in the article referred to, is said to use a gallon of crude petroleum per HP. per day of twelve hours, this amount being slightly exceeded in engines of less than 3 HP. The Hock engine is stated to use what is equivalent to 2.75 pounds of naphtha per hour per HP., at the cost of about 3*d.* per hour, a 1-HP. engine being in first cost about £50. Petroleum motors may be advantageously used in place of gas engines where a

* "Engineering" vol. xxiii., p. 127.
† "Klein-Kraft-Maschinen," p. 32.

supply of gas cannot be obtained, and where petroleum is abundant, but this leads to the conclusion that, although in America and in Germany their use will probably increase, in this country, at present, it is not likely to do so.

PNEUMATIC PROPELLER FOR RAILWAYS.

By L. GONIN.

From "Abstracts" of the Institution of Civil Engineers.

THE author describes a new form of valve and piston carriage for the tube of a pneumatic railway. It is proposed to use air compressed to 6 atmospheres, and to apply the system, on grounds of economy and safety, to lines having heavy gradients and but little level track. The tube is of cast iron provided with numerous exterior stiffening ribs, and fixed upon cross sleepers between the ordinary rails, the latter being laid on longitudinal resting on the transverse sleepers. The slot in the upper part of the tube is beveled on its interior edges, thus forming a seating for the continuous valve. The valve, made of greased leather, wrapped round a wooden body and having an iron band above and below, is in the form of a truncated wedge with the narrow end upwards, so that the pressure of the air within the tube may press the valve tightly into the wedge-formed sides of the longitudinal slot. The valve, when not pressed upwards by the air, hangs within the tube from a continuous flat band of iron, wide enough to overlap the slot in the tube, and to rest on the edges of the slot. Both iron band and valve are sufficiently flexible, their connection at a distance of some inches by bolts not materially affecting this quality. When the valve is in mid-position, vertically, so that the valve is clear below its seat, and the iron band is clear above the upper edge of the slot, there is room on each side of the connecting bolts for the passage of a double coulter piece, connecting the piston in the tube with the carriage above, which is driven by it. The part of the valve at and behind the piston is kept up in its highest position by the compressed air, while the part ahead of the piston falls to its lowest position. The carriage driven by the piston carries in front a small wheel which runs upon and presses the iron band down, and thus regulates its position for the passage of the coulter. The coulter may be either attached to a large vehicle or, by the author's preference, to a small chariot running on separate rails on the top of the tube, and drawing its load by chain connection, or the load may be pushed by a buffer. The corresponding buffer, which would be carried by the engines on a line employing the compressed air system for helping trains up heavy gradients, is made to rise and fall so as to clear the chariot until beyond it. It is then dropped, and the chariot brought into work. The arrangement of valve described differs entirely from those of Clegg and Samuda, employedon the atmospheric and pneumatic railways,and it affords a tight joint, while it has not to be raised by, nor is it exposed to any wear by, the coulter. The latter is moreover double, one-half passing into the tube on either side of the valve, the strains upon it being thus of a simple order.

As the quantity of power required for compressing air does not increase with the increase of pressure, but in proportion to the logarithm of the pressures, the author proposes pressure as high as from 6 to 10 atmospheres, so that the air may be employed expansively, the full pressure being used for starting and traveling the greater part of the journey on inclines; the air would then be cut off, and the remainder of the journey performed by expansion. An experimental length of tube, 10 inches in diameter and 131 feet in length, has been constructed, the results of experiments with which are given.

NOTE ON THE PRESENT SYSTEM OF PERMANENT WAY ON THE SIX CHIEF RAILWAYS OF FRANCE.—By E. Lecocq.—Steel rails are now almost exclusively used, but the section and weight of rail differ widely. The Eastern, the Northern, and the Paris, Lyons, and Mediterranean companies use only the flange rail; but the Orleans, the Midland and Western use also the old double-headed rail. The flange rail used on the Northern, and on recent extensions of the Western lines, weighs 60½ lbs. per yard, and is noticeable chiefly for the small width of the base (3 13/16 inches), as compared with the height (5 inches bare). This type of rail was introduced on the Northern some six years ago, so presumably it has proved satisfactory.

The Eastern company uses also a 60½ lbs. rail, but the height is reduced to 4¾ inches, and the base increased to nearly 4 inches.

The Lyons company employs two sections, the heavier one weighing 77½ lbs. per yard, and measuring 5⅛ inches high, by 5⅛ inches wide, and the lighter one weighing 66½ lbs. per yard, and measuring 5 inches by 4 inches base.

The width of head as a rule is 2¼ inches, and the length of rail about 26 feet. With the exception of some practically obsolete types of "pear headed" rail, the angle for fishing is ½ to 1, and the thickness of web in most instances is rather less than ½ inch.

A diagram and table, illustrating the arrangement of the sleepers and joints, lead to the following conclusions:

1. Suspended joints are adopted by five out of the six companies, the Northern alone putting a sleeper under the joint.

2. The spacing of the sleepers next the joint is always about 2 feet from center to center.

3. On the Lyons and on the Western railways, the sleepers next to those at the joint are spaced a less distance apart than the remainder. This also is necessarily the case on the Eastern and Northern lines, because the fish-joints are not opposite each other, but lap about 2 feet.

4. Expedients to prevent the longitudinal movement of the rails are adopted by five out of the six companies.

5. Finally, it appears that, with the exception of the Lyons company, the weight of the flange rail track is about 323 lbs. per yard, while the double-headed way attains a mean weight of 403 lbs. per yard. This excess of 80 lbs. per yard is considered by the author to be eminently favorable to stability, especially with the heavy loading and great speed of modern trains.

The following are matters of details not shown in the table, but considered worthy of notice:

1. Four bolts are now invariably used for fishing, instead of three, as was commonly the case some time ago.

2. Grooved fishes are used by three of the companies; the others use shoulder bolts.

3. Wood screws are exclusively used for fastening the rails and chairs by the Eastern, Northern, and Western companies, and by the Lyons company for their lighter rail. Trenails are still used for fastening the chairs by the Southern and Orleans companies. The Lyons company fasten their heavy rail with a fang bolt on the outside, and a trenail on the inside of the 5⅛ inches flange.—*Revue générale des Chemins de fer.*

IT is well known that ordinary letter paper if rubbed acquires electric properties. M. Wideman, has, however, says the *Electrician*, found that if one takes Swedish filter paper, or this paper laid between pieces of letter paper, and subjects it to the following treatment, it displays strong electric properties, and sparks several centimeters in length can be obtained: The paper should be plunged into a mixture of nitric acid and sulphuric acid of equal volume. The paper thus pyroxilized is then washed in plenty of water and dried. Then rub quickly, having stretched it on a waxed cloth, in order to give it electric properties. Nearly every experiment with static electricity can be accomplished with the paper.

THE people of Portage La Prairie have consented to make a branch line to the Canada Pacific Railway of five miles in length, if the Government would furnish rails and rolling stock, and to hand it over to the Government on completion.

NOTES ON RAILROADING.

By J. A. L. WADDELL, C. E.

From the Papers of the Pi Eta Society of Rensselaer Polytechnic Institute.

THE subject with which I beg leave to occupy your attention this evening and, if such be your pleasure, at some future meetings also, is that of Railroading. It is not my intention to cover in these notes the whole subject of Railroad Engineering, but simply to convey to you, who are soon to become engineers, an idea of what *practical* railroading really is, in order that you may not feel at a loss how to proceed should you, at any time, be called upon to take charge of a survey or construction party. The facts here stated have been obtained principally from personal observation and experience, and I have avoided as much as possible dealing with those parts of the subject that are treated in standard works; nevertheless for the purpose of making what I have written clear and continuous, I have had to touch lightly upon some of them. In dealing with such a technical subject as this one, it is impossible to avoid repetition of certain words and expressions, so you will please make all due allowance for the phraseology of what I am about to read to you, and do not expect to hear the flowing language and easy style which one might employ in dealing with a literary subject. Especial reference is made to "bush work," for by far the larger portion of American railroading partakes of that character; besides, nearly all the difficulties met with in running a line through a well-settled country or prairie land are encountered in the bush, in addition to many others of an entirely different nature. The building of a line of road may be divided into four distinct steps, which will be treated separately; they are Exploration, Preliminary, Location and Construction.

EXPLORATION.

The data usually given to the engineer who makes the exploratory survey, are the *terminii* of the line, various points through which it is to pass, and the general character of the road. The latter is determined by the principal object for which the road is to be built, whether for passenger or freight traffic, and, if for the latter, whether it is to be heavy or light. This influences the grades, and, therefore, also the length and cost of the road. The heavier the traffic, the more expensive will usually be the first cost of construction, for hills will have to be avoided or cut through, that would otherwise have been run over, thus augmenting the expense by reason of either the increased length of line or the greater quantity of material to be moved. On most roads the heavy freight goes in one direction, so in that direction the upgrades should, if possible, be the lightest. Another reason for having two maximum grades is that, in case a summit is to be reached, the engineer, by having a light maximum down grade in the direction of the line, will be prevented from making the profile too irregular. Having given, then, the *terminii*, intermediate points and perhaps also the maximum grades, an exploratory survey is made by an experienced engineer passing over the country, so as to report whether, in his opinion, a practicable route can be found, and, if so, what would be its approximate location. If the character of the country permit, he travels on horseback; though usually, it is necessary to go on foot, in which case he is accompanied by one or two packmen. His equipment need consist only of an axe, a pair of field glasses, a note book, a hand level or two barometers and a pair of steel climbers to enable him to climb a tree with facility. By means of the latter he will often be able to obtain a good knowledge of the country, and to take such general observations as may be of use in the preliminary survey. He must estimate distances and elevations, or obtain the latter by means of the barometers or hand level, make notes as to the course of rivers and streams and their crossings, establish the general direction of the chains of hills, locate passes, look out for water communication or some other way to obtain access to the differ-

ent parts of the line, so as to facilitate the forwarding of supplies, and, in short, acquire the greatest amount of knowledge of the country in the least possible time. Upon his report depends the

PRELIMINARY SURVEY,

the party for which should consist of a chief, a transitman, two levelers, two rodmen, a topographer, one or two picketmen, two chainmen, five axemen for the line, and one for each leveling party, three to six or even eight packmen, and sometimes a commissariat officer and an explorer.

As in most preliminary surveys, the duties of any one of these officers are substantially the same. The instructions of the chief engineer of the Canadian Pacific Railway to the Staff may be taken as a general illustration of their character. The gentleman placed at the head of a party is required to take general charge of it, and is held responsible for the execution of all instructions, as well as for the maintenance of proper discipline in the party. Every member of the party is under his charge and must obey his orders. In conducting the survey, he is expected to be at its head every day, exploring in front and to the right and left of the line, in order to see what obstacles may be in the way of the same, and, if serious, decide as to the best manner of avoiding them. When he finds it necessary to leave the party, or in the event of illness, he should nominate the person to act in his place for the time being; in the event of his failing to do so the transitman should take charge.

The duty of the transitman is to run the instrument, keep the notes, and in case there is no topographer, to take the topography.

The duty of the first leveler is to make a profile of the line and cross sections; that of the second leveler is to check the bench marks and to assist in cross-sectioning.

The principal duty of the others of the party is to obey all orders with diligence and to the best of their ability.

Before starting the survey the transit should be set up over the terminus and a true north and south line determined by means of an observation of the pole star. To do this, place the instrument at zero, choose some well-marked point, set the cross hairs upon it a little before dark, and see that no one disturbs the transit between that time and the time when the observation is to be taken, the hour for which can be found in the Nautical Almanac. It is usual to observe the star when at its greatest eastern or western elongation, but it can be observed when at its maximum depression below or elevation above the pole, when the instrument is provided with a tangent screw to produce a slow motion in a vertical plane. In the latter case no correction is required, so when the star has reached its greatest elevation or depression, which can be ascertained by keeping the intersection of the cross hairs on the star, being careful never to pass it, and noting when it appears to cease rising or falling, all that is necessary is to take the reading of the horizontal circle, which will give the angle that the line from the terminus to the point chosen makes with the true north. In observing the greatest elongation, keep the vertical hair on the star and note when it appears to cease changing its azimuth, then take the reading, which will be the angle that the above-mentioned line makes with the vertical plane through the star at its greatest elongation. The correction for this angle can be found by a very simple rule given in the Nautical Almanac, and is to be added, if the star lie between the pole and the line, and to be substracted if it does not. Should the line lie between the pole and the star, this will give a negative result, which shows that the angle so obtained is to be laid off from the pole towards the star; *i. e.*, if the elogation be east, the bearing given by the minus angle will be east. It is easily seen that the latter method is by far the better, for it is necessary to use only the horizontal tangent screw (the one which gives the upper motion, of course), while in the former both the horizontal and vertical screws have to be turned at the same time. Having found, then, the true bearing of a fixed line, the true bearing of the initial line of the survey can be easily ascertained by measuring the horizontal angle between the two lines.

Suppose the party all ready to start and the initial bearing taken. The axemen go ahead clearing the line and the

chainmen follow, driving stakes about three feet long and two inches wide firmly at every hundred feet, numbering them consecutively, with red chalk as they go. When it is necessary to change the instrument, either on account of the distance of the picketman, who has been keeping the axemen on line, a sudden rise or fall of the ground or a desired change of direction, the transitman gives the signal previously agreed upon for "hub," the picketman chooses a convenient spot for setting up and finds "point" on the line into which one or two of the axemen drive a stake from four to six inches in diameter, and from one to six feet long according to the quality of the ground, and the picketman finds "point on hub" into which he drives a nail.

The transitman then sets the back picket, which in bush work is a small straight sapling whitened on one side, with a cross-piece eight or ten inches long, held in a cleft near the top, picks up his instrument, moves on to the new hub, over which he sets up, sights to his back picket, reverses his instrument and starts the axemen and chainmen to work. Before giving hub again, the transitman should sight once more to the back picket in order to satisfy himself that his instrument has not been moved, and if uncertain of the adjustment should turn the instrument round 180° by means of the lower movement, sight to the back picket, reverse and give "second point on hub." If this coincides with the first one all right, if not, then the distance between the two (measured, of course, at right angles to the line) must be bisected accurately and the nail be driven at the point of bisection. This is, in fact, performing the second adjustment of the transit. Beside each hub, at a distance of about three feet, should be driven a large stake called "hub stake," five feet long, at least five inches in diameter and squared at the top. On it should be written the *exact* chainage of the point on hub, and the distinguishing mark of the survey party, which is usually one or two letters. In setting up in swampy ground, "legs" or long heavy stakes should be driven flush with the surface of the ground, in order to provide a tolerably firm "set up" for the nstrument. The plumb bob is brought *nearly* over the nail by driving down first one leg and then another, then moving the feet of the tripod until the point of the plumb-bob exactly covers the nail. If the instrument having a shifting head much time and trouble will be saved. Hubs should be placed at the summits and bases of hills, so as to avoid as much chopping and moving of the instrument as possible. At intersection points the hub-stakes should be marked "apex," and the angle to right or left should be written on another of the squared faces. The best kind of a hub is the stump of a tree that comes directly on line, for it cannot be knocked out of place. In running through comparatively level country it is well to keep as much as possible to the low ground without lengthening the line greatly or making it too crooked.

It is to be remembered that the survey is not a final one, so that it does not pay to turn out for a small ridge, but to go over it and take a cross section on the summit to where it tails off. In case of trying to reach a summit, it is necessary to follow the contours of the hills so as to obtain a gradual rise. In hilly country where the line rises and falls in long stretches, the best way to proceed is to follow up a stream to near its source, then cut across the divide and follow down another stream on the other slope. If the banks of a river are high, a good crossing can often be found by going up stream, where they are usually lower. In crossing a lake on the ice soundings should be taken, as it might be found desirable to drain the lake.

Offsets usually are taken at right angles to the line, but sometimes inclined. Short ones can quickly be turned off by standing on the centre line, stretching the arms at full length along the line, then bringing them suddenly together; with a little practice a right angle can thus be turned quite accurately. For long distances it is necessary to use an instrument of some sort, a cross head being as good as any. Offsets are needed to locate the bases of hills and the edges of streams. Traverses of ridges or rivers are most easily made with the compass, provided there is no local attraction, by setting up at every alternate station and reading the bearing of both lines intersecting at that point. The compass is sometimes used on the main line in level

country where there is no great local attraction. Very often there is no instrument used at all, except at the angles, the long tangents being run in by pickets. It is surprising with what accuracy work can be done by the compass and pickets in a comparatively level country. Running a picket line in the bush is quite a different affair from what is taught at the R. P. I. during the chain survey, it may therefore be well to describe the operation: The pickets are thin straight saplings, about five and a half feet in length, brought to a long fine point at the top, and sharpened at the lower ends for pushing into the ground. The tops of two of them are set on line by the instrument about fifty feet apart; a third is then set about fifty feet farther on, the top being brought exactly on line with the others. A check on the work can always be obtained by glancing back along the line and ascertaining if the last three or four pickets are in one straight line. A pair of field glasses is of great assistance in picketing. To "tie on" to an old line, set a hub on each side of it, drive a nail in each along the line of sight of the instrument, and stretch a string tightly between the nails. Next take up the instrument, set it up on the old line, and in its direction, and sight to a picket moved along the string until covered by the vertical hair; then drive a hub and set a nail at the point thus found.

After the transit party has made a good start, the levelers commence work; they should never approach the transit closely enough to disturb the back picket. If the second leveler finds a difference of more than one-tenth of a foot between benches, the levels should be rechecked. The speaking rod is almost exclusively used in bush work; it is much better than the target rod for rapidity, and the leveler is not at the mercy of his rodman, who may often be too ignorant to read the rod. Care should be taken to equalize back sights and fore sights, and neither should exceed for accurate work four hundred feet. It is possible to read an ordinary rod on a clear day at eight or nine hundred feet, but one cannot rely on levels so taken. The best kind of level for rapid work, as far as my experience goes, is the Pastorelli. It is fourteen inches in length and has a ball and socket joint, by means of which it can be set up in places where it would be impossible to use an ordinary Y level. The telescope being an inverting one may cause a little trouble at first, but it does not take long to become accustomed to that. The best kind of rod is a piece of well-seasoned pine wood sixteen or eighteen feet long, four inches wide and one and a-half inches thick at bottom, three inches wide, and three-quarters of an inch thick at top, and divided into tenths and half-tenths. The form of level book should be that shown in the figure below:

Sta.	B.S.	Int.	F.S.	H.I.	Elev.	Remarks.

The F. S. column is for *turning points alone*, so that by adding up the B. S. and F. S. columns, subtracting one from the other, and comparing the remainder with the difference in elevation of the starting and final points, a check on the accuracy of the book can be obtained. Bench marks should be made about every fifteen hundred feet, and should be well defined so as to catch the eye readily. The ordinary method of making them is to blaze the side of a tree or stump, drive a nail into a projecting knob on the root and write the elevation of the top of the nail on the blazed part, together with the distinguishing mark of the survey party. The exact location of the bench should be noted in the level book in the column for remarks. To make a good bench mark without the use of a nail, slice off the side of a stump to very near the bottom, and bring the remaining part to a rounded apex.

The leveler should note every stream and river crossed, its size, direction, level of surface, difference of levels between high and low water, velocity of current, probable discharge, and any peculiarities which it may seem to possess. While in the field it is not necessary to work out the elevations of intermediates; it is sufficient to know the height of instrument

and the elevations of bench marks. Every evening the leveler should "make up" his book and plot the profile. Leveling can be done very rapidly in winter, as traveling on snow shoes is so much easier than ordinary walking in the bush. The uncertainty as to the shape of the ground beneath the snow makes it sometimes difficult to get a good set-up, but if the three legs of the tripod be kept vertical and the snow be packed well about them, the instrument will be kept as steady as in any ordinary set-up. If the rodman carry a three-foot stake to be pushed through the snow to the ground at intermediates, and upon which the rod is placed, much time may be saved. In this case care should be taken to add three feet each time to the reading of the rod. The transit book ordinarily used has one page ruled into squares, each side representing one hundred feet, and the other page ruled for remarks, a red line running through the center of each. In case there be a topographer in the party, the transitman need make note only of the chainage, magnetic bearings and deflection angles, being sure to state whether they are to right or left. But if there be no topographer, it will be necessary for him to keep full topographical notes. These consist of offsets to right and left wherever necessary, contours of hills, courses of streams, observations as to the quality of timber and soil, location of ridges and lakes, etc., etc.

The first entry in a note book each morning should be the name of the person acting as transitman, leveler or topographer, as the case may be. All notes should be clearly and distinctly made in pencil on the spot; no additional notes should be entered with the original notes after the day on which the latter are written. Field notes should not be inked or changed in any way; copies of them may be made in ink and reduced levels entered in ink. This is to prevent what is called "cooking" the notes, an offence of which no true engineer is ever guilty. Any man found tampering with the level notes or benches, or trying any scheme to make poor work pass for good, gets his discharge without any compunction on the part of the chief engineer or those who are interested in seeing work well done. A plot of the day's work should be made every night and the latitude and departure worked out. Checks on the work should be made as often as possible. The check for deflections is made by taking an observation of the pole star every ten or fifteen miles and comparing the reckoned true bearing with the observed, after making the proper correction for conveyance. This is found as follows: Let n= the number of miles in a degree of longitude, a quantity depending upon the latitude of the place, then will $60 \div n$=the correction in minutes per mile of longitude. Calling m the number of miles of longitude between the two points considered, $60m \div n$ will be the required correction. A check for the distance can be roughly obtained by ascertaining the latitude and longitude of various points along the line, by observations of the stars. Checks can sometimes be obtained by tying on to old lines. If an error be found in the direction of the line, it may have been made by recording an angle to the wrong side; to ascertain if such be the case, divide the error by two and see if there be an angle on the plot which is equal or very nearly equal to this result. If there be one the chances are that the mistake was made in that way. In case the error cannot be located, it will be necessary to go over the work again, commencing at the point where the next to last observation was taken. Errors are sometimes made in reading a deflection angle, and sometimes by making a crook in the line owing to the transit being out of adjustment or level. Whenever an observation of the pole star is made, the variation of the needle should be noted. As the survey progresses, the engineer in charge should project a location line on the map by means of the cross sections taken to right and left, in order to give a general idea of where the true location of the road will be. He is expected to keep a diary in which to note the progress of the work each day, the difficulties overcome, and everything relating to the survey. He should see that the camp is properly supplied with everything needed, and should, if possible, have depots for provisions made at points near which the line will pass, so as to avoid packing them over the line. Camp should be moved every three or four miles, in order to avoid long walks

to the place of starting work in the morning. It may be necessary during the progress of the survey to forward special instructions to the engineer in charge; for this and other reasons, that officer should take especial care that whenever the camping ground is changed, a notice be distinctly written upon a tree, or in some other conspicuous position, containing the following information:

1. The distinguishing letter of the survey.
2. The number of the camp.
3. The date of removal of camp.
4. The probable direction and distance to the next camping ground.
5. The name of the engineer in charge of the party.

The success of the survey is materially assisted by each man endeavoring to save as much time as he can. A few hints as to how this can be effected may be of use. Before moving the instrument the transitman should, if he wish to prolong the line, set two pickets on line *beyond the new hub,* in order that the axemen near continue their chopping while he is moving on and setting up. He can set his own back picket, if it be left ready for him, by placing it close behind the instrument and bringing the cross to the level of the eye-piece. As soon as point for hub is called for, one of the axemen should commence making a hub, and if the ground require it, legs for the instrument. The fore chainman, who carries a light axe, should occupy himself in cutting stakes when not busy chaining.

There is a great variety of opinion as to what transit is the best for bush work. Some engineers prefer those of English manufacture, others those made in this country. The most satisfactory that I have ever tried is one of Gurley's manufacture.

LOCATION.

After the results of the different preliminary surveys (if there be more than one) have been handed into the office, it is there decided what is to be the approximate location of the line. The party for location is the same as for preliminary, except that there is no explorer needed. The general style of the work is the same, the principal differences being that, as the survey is a final one, the centers are run in on the curves, and greater care is taken in getting the elevations of points between the stakes and on the cross sections. The subject of curves is so well treated in Henck's Field Book for Engineers that it is useless to go into it here. It will be sufficient to state that the formulæ most often employed are $T=R\tan\frac{1}{2}I$ and $C=R\tan\frac{1}{2}I\tan\frac{1}{4}I$, where T is the sub-tangent, R the radius of the curve, I the angle of intersection and C the crown distance. The latter is used as a check for the correctness of the first half of a curve, which is so long as to require several changes of instrument. The number of stakes put in at one setting-up depends altogether upon the natural features of the ground, the degree of the curve and the size and thickness of the timber. In running around a rocky bluff it may be impossible to see more than one hundred feet at a time, especially if the angle of deflection be large; in which case it would be well to put in stakes fifty feet apart.

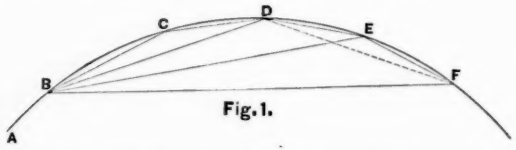


Fig. 1.

In Plate I., Fig. 1, suppose the transit to be set up at B to run in the curve BCDEF, &c., from the initial tangent AB. If the points C, D, E and F are all to be located from the point B, it will be necessary to clear out all the timber on the area included between the chords BC, CD, DE, EF and FB. Now if the curve be an easy one, that is of light curvature, say thirty minutes, that area will not be great, so that it would take less time ordinarily to do that amount of chopping than to make a new set-up at D, in which case the areas of the triangles BCD and DEF only would have to be cleared. If the curve were heavy

it can easily be seen that time would be saved by making the two setting-up. Before commencing the chopping, the transitman decides how many stations he will lay off and sets a picket on each extreme chord (BC and BF in the case taken above) at a short distance, say twenty feet from the instrument, so as to serve as a guide for the axemen. One slight difference between laying out a curve in the field and the explanation usually given in text books for doing it, should be noticed. In text books the first chord is taken to be one hundred feet, while in practice it very rarely occurs that *a* B.C. (beginning of curve) comes at an even station, so that the first chord is nearly always less than a full chain, and the first deflection angle will be equal to the difference between the chainage of the B.C. and that of the next full station divided by one hundred and multiplied by one-half the degree of the curve; for example, if the chainage of the B.C. is 1021+54.6, and it is a three-degree curve, the first deflection angle will be $\frac{1022-1021+54.6\times 1^\circ 30'}{100}$ $=\frac{45.4}{100}\times 1^\circ\ 30'=0^\circ\ 40'\ 52''$ nearly, or in practice 0° 41′. The second angle of deflection will be 2° 11′, the third 3° 41′ etc. If the length of the curve be 925 feet (=100 I÷3°), the chainage of the E.C. (end of curve) will be 1030+79.6, the last chord will therefore be 79.6 feet long and the corresponding deflection angle, or increase of deflection angle, as the case may be, will be $\frac{79.6}{100}+1^\circ\ 30'=$ 1° 11½′. By 1030+79.6 is meant 1030 chains and 79.6 feet; it is always written thus on the hub stakes. The degree of a curve should be made as small as possible, for the longer the curve the shorter the line; the "lay of the ground" nearly always decides the degree of curvature to employ in connecting two tangents, that curve being chosen which will give the best profile, or in case of a choice between rock and earth excavation, *the least expensive line*, the limiting value of the degree of curvature, of course, being never exceeded. This limiting value was decided upon at the same time as that for the grades. Where a grade and a curve occur together, a resistance to motion is caused by each, so to make the effect not exceed that due to the maximum grade on a straight line, the limit of the former has to be changed. It has been found experimentally that a one-degree curve causes as much resistance as a 2.4 feet rise per mile, so that the limiting value for grades will have to be reduced by that amount for every degree of curvature. The value 2.4 was found for a speed of twenty miles per hour.

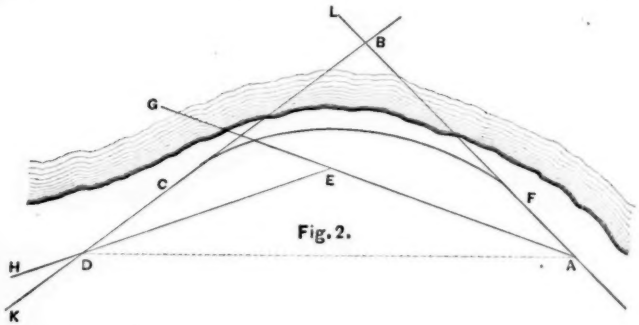


Fig. 2.

Locations are sometimes made by taking the angles and distances directly from the plot and laying them out upon the ground, but the ordinary way is to use the plot simply as a guide, and to run in the tangents and put in curves to fit. In going around a rocky shore the intersection points often fall in the water, as shown in Plate I., Fig. 2. To run in the curve F C, run the random lines A E and E D, measuring the angles B A E=i, G E D=i_1 and H D K=i_2 then L B D=I=$i+i_1+i_2$. In the triangle E A D, E A, E D and the angle A E D are known, so that D A and the angles E A

D and E D A can be computed. This will give the angles B A D and B D A, so that in the triangle B A D, the sides B A and B D can be found. After deciding upon the degree of the curve, the sub-tangent can be calculated in the ordinary manner, the differences between this distance and the lengths A B and B D be laid off from the points A and D, and the curve run in as usual.

If several location lines have been run in the same neighborhood, the stakes on the one adopted should be painted, so as to readily catch the eye.

CONSTRUCTION.

The first steps to be taken after the final location has been decided upon is the clearing of the right of way. The width of clearing should be about one hundred and thirty feet, in order to allow for high banks and wide side ditches, and for the erection of a telegraph line, which should be placed on the berme, or space between the foot of the slope of the bank and the side ditch, and always on the same side of the line. As a wide clearing is necessary to prevent trees from falling on the wires, the center line of the road should not pass through the middle of the clearing, but far enough to one side to allow the telegraph line to occupy that position. Assuming the average fill to be three feet, the top width of bank to be seventeen feet and the slope ratio of the sides to be one and a-half horizontal to one vertical, this would make the slope stakes extend thirteen feet from the center line, so if the latter be fifty feet distant from one side of the clearing and eighty feet from the other, the telegraph line could occupy the center. If by reason of a high bank or a deep cutting the telegraph line be brought too near one side of the clearing, the latter should be widened there; and in any case, all trees which by falling, would strike the wires should be cut down. The trees felled on the clearing should be winrowed and burned, and any logs left by the fire should be hauled into the bush or be burned a second time. After the line has been properly cleared, *and not until then*, should the section engineer commence operations. His first duty should be to lay out the off-take drains, for full directions concerning which, and railroad drainage in general, see an article on the subject in the "Pi Eta Pamphlet" of 1878–9.

He should next re-establish the center line, for most of the stakes and perhaps some of the hubs will have been destroyed during the clearing. While doing this he should reference all hubs at the beginnings and ends of curves, and on tangents at least one in every two thousand feet, and at apices in the grade, in order to facilitate the finding of the center line after the grading is finished. The ordinary method of referencing is to turn off an angle of 45° to the tangent, set a hub at 100 feet, driving a hub stake beside it with R. H. marked thereon, reverse the instrument and place another in the same manner; then turn off an angle of 45° to the other side of the tangent and proceed as before. Should the natural features of the country or any other cause prevent the setting of hubs on both sides of the line, it will be necessary to set two hubs on each reference line, as shown in Plate I., Fig. 3. Care should always be taken that the hubs are placed in positions where they would not be liable to be disturbed.

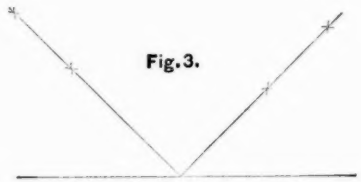


Fig. 3.

I have employed the following method and prefer it to any other, although it was not in accordance with the instructions given to the section engineer of the road on which I was working: Set up the instrument over the hub to be referenced, turn it until two trees are found on line with the hub, and cut down the most convenient one close to the ground, so that the instrument can be set up over the stump. After having clamped the plate, set a nail on the stump, then reverse and set another on a blaze upon the other tree. Turn the instrument at about right angles to this reference line, find to other trees on line and proceed as before. By this means the hubs are set so permanently that even fire will not destroy them. Their

positions should be accurately noted in the field book.

The next step should be to to run over the levels very carefully, placing benches at every thousand feet on the edge of the clearing most distant from the center line, so as to be out of the way of the graders. Benches should be always placed at both ends of heavy cuttings for easy reference in driving grade plugs, during the progress of the work. Where the line does not pass through wooded country and no convenient object can be found for a bench mark, one should be made by sinking a post in the ground five or six feet in order to be below frost, and marking the elevation of a nail driven in the top of it upon a stake placed alongside. Plugs two inches in diameter and six or eight long should be driven at the foot and directly in front of each center stake. They should be made level with the service of the ground, and upon them the leveling rod be held when levels are taken. Should the plugs not be driven to the surface, it will cause a large discrepancy between the actual quantity of material excavated and that called for by the profile. If the section engineer would like to avoid all chance of receiving an overhauling from the office, on account of such discrepancy, he can do so by clearing away from the foot of each stake all moss and spongy material and driving the plug down to solid ground. In this way the true level of the base of the embankment would be recorded, for all the moss, etc., is compressed to almost nothing by the weight of the superincumbent earth. It may be unnecessary labor, but it is well to look ahead a little, especially if those in charge of affairs are at all inclined to be unreasonable.

After the profile of the line has been made on profile paper to a scale of 200 feet to the inch horizontal, and 20 vertical, the gradients should be carefully established. The objects to be attained in fixing them are an equality of cuts and fills to as great an extent as is judicious, and the avoidance of long hauls and irregularities in the continuous grade line.

The next thing to be done is to write the cuts and fills on the center stakes, at the same time setting the slope and ditch stakes. To set the former when the ground does not slope greatly, add together (for earth cuttings and embankments) one-half the width of road bed (ordinarily 11 feet for cuts and 8½ feet for fills), the center cut or fill and half the latter quantity; measure off the distance so found at right angles to the center line, *estimate* the rise or fall in that distance, add to or subtract from the distance three halves of the rise or fall and hold the rod at this new distance. The level having been previously set up and the height of instrument above grade determined, sight to the rod, subtract the reading from the latter quantity, and add to it one-half the width of road bed and one-half of the quantity itself. Should the sum thus obtained equal the distance of the point, where the rod is held, from the center stake, set the slope stake there; if not, try again until the two quantities agree within two or three tenths of a foot. Set the ditch stake outside of the slope stake at a distance equal to the width of berme, which varies from six to fifteen and even twenty feet, according to the stability of the material, the most common value being ten feet. On the slope stakes should be marked *s. s.*, and their distance from the center line; the ditch stakes should be marked D. In grading side ditches it is a good plan to drive a plug four or five feet outside of the ditch stake and mark on the latter the cut below the former.

At the same time that the slope stakes are being set, elevations should be taken on every cross section at distances not equal to the semi-width of road bed, and wherever the slope changes; they should be taken, too, at some distance beyond the ditch stakes. Cross sections should be taken and slope stakes set wherever the slope of the ground or the grade changes. Cross sections should be set out instrumentally at right angles to the center line on tangents; and on curved portions at right angles to the tangent of the curve.

The levels on the cross sections should be taken in the same manner as those on the longitudinal line, commencing on a bench mark, and using the same datum; they should be continued through a series of cross sections till another bench mark is reached, on which a level should be taken to test the accuracy of the work.

Grade plugs should be driven at the ends of cuttings, where the grade cuts the natural surface, both on the center line and at a distance on each side equal to the semi-width of road bed in cuttings. Driving a grade plug where the grade is level is quite an easy matter, for all that is necessary is to make the rodman hold his rod, at different points, until the rod reads an amount equal to the difference between the height of instrument and formation level: but when the grade is a rising or a falling one, the change in the height of formation level must be taken into account every time that the rodman moves the rod in the direction of the line. It is sometimes customary to drive, at the ends of large cuttings, grade plugs two or three feet long and as many inches in diameter, the upper end being bound with an iron ring so as to be useful as a rough reference during the course of the work. The form of level book used on construction differs somewhat from that used on preliminary, and is given below.

Sta.	B. S.	Int.	F. S	H. I.	Elev.	Grade.	Cut.	Fill.	Remarks.
.....	3.26	..	..	1128.30	1125.04	..	..	..	B. M., No. 23.
1320+55...v...	..	11.21	..	..	1117.09	1116.55	0.54	..	Grade 1 per 100.
R. 11.....	..	10.20	..	..	1118.10	1116.55	1.55	..	....
R. 14, s. s.....	..	9.80	..	..	1118.50	1116.55	1.95	..	....
L. 8.5.....	..	13.20	..	..	1115.10	..	..	1.45	....
L. 13.5 s. s.....	..	14.40	..	..	1113.90	..	..	2.65	....
L. 30.....	..	16.80	..	..	1111.50	..	..	..	....
T. P.....	..	..	0.28	..	1122.02	..	..	..	....

The notes here given show the center levels and the cross sections taken together, as would be the case were the benches run in by flying levels. In setting slope stakes in swamps it is not necessary to use the instrument: simply set them at the distances called for by the fills marked on the center stakes, allowing a little for the subsidence of the surface, if it be thought advisable.

Before commencing to work a cutting, the quantity of material in it and its quality should be determined, so as to decide how much should be carried each way, and to ascertain how far it will complete the embankment at each end, due allowance being made for the shrinkage of the material. The ground is now ready for the contractor to set the graders to work.

As stated in the article in the Pi Eta Pamphlet, already referred to, he should commence operations at the ends of the off-take drains and continue them across the right of way to the opposite side ditch, in order to start four gangs of men in each large depression of the line to build the embankment from the side ditches. While the embankment is being built, both the contractor and engineer should keep an eye on the graders, especially if they are doing piece work, so as to see that the close cutting and grubbing are properly attended to, and that no poor material, such as roots, moss or logs, is thrown in the bank. The last, unless very large, should be kept higher in the middle than at the sides all through the progress of the work, and when finished the upper surface should be rounded off so as to fall from four to six inches from center to sides. It should be left considerably above grade, so as to allow for subsidence, the amount being determined by previous experience.

By the term "grubbing" is meant tearing the stumps out by the roots, and by "close cutting," chopping them down nearly level with the ground. Grubbing occurs on the off-take drains, on side ditches, in embankments under two feet in depth, and in line cuttings, though it is not often paid for where the cutting exceeds three or four feet, or in borrow pits. Close cutting should be done in enbankments over two and under five or six feet in depth. A safe rule to go by is, "never to allow a stump to reach within two feet of grade," for the latter may some time be lowered if the bank should sink uniformly below the original grade. It costs more to chop one pro-

jecting stump out of the bank than to close cut a dozen before commencing the grading.

Before opening an earth cut the engineer should see that the side ditches are finished as far as the embankment will extend, and that the material excavated from the former is cast into the latter. In shallow cuttings it is sometimes necessary to excavate below grade in order to remove all the loose surface material, and to fill either with ballast or firm earth taken from a deeper portion of the cutting or elsewhere. All large stones and boulders measuring less than 27 cubic feet, and all loose rocks that may be removed with facility by hand, pick or bar, without the necessity of blasting, are classed as "loose rock." All stones and boulders measuring more than 27 cubic feet, and all solid quarry rock are classed as "solid rock." In cuttings composed of both earth and solid rock, a berme of six feet is to be left on the upper surface of the latter. Owing to its removal, earth shrinks from eight to twenty-five per cent. of its volume, loose rock changes but little and solid rock occupies one and a half times the space in embankments that it did in its natural state. Earth cuttings and embankments have slopes of one and a half horizontal to one vertical; rock cuttings, one horizontal to four vertical, and rock embankments one to one. Where the side ditches and line cuttings do not furnish enough material for the banks, borrow pits are rendered necessary. To lay one out, fix a line far enough from the pit not to be disturbed by the excavation, and divide up the surface of the ground that is to be removed into squares, whose sides are parallel and perpendicular to the line of reference, then take the elevations of the corners of these squares above the datum used on the line. After the borrow pit is finished, re-locate the corners of the squares and take their new elevations. This divides the volume removed into vertical square prisms, whose length can be found by averaging the differences of the two elevations taken at each corner. If the length of the sides of the squares be taken equal to some multiple of three feet, the calculation of the cubical content will be greatly facilitated. Thus, if the length of each side is thirty feet or ten yards, the horizontal section is one hundred square yards. If the average of the differences of elevations at the four corners of any square is twenty-seven feet or nine yards the cubical content of the prism will be nine hundred yards. Station grounds should be laid out, if possible, on long tangents where the grade is nearly level; or at least does not exceed nineteen feet to the mile. If it is necessary to place one on a curve, let the station house be situated near the apex, to allow the station master to see trains approaching both ways. Clearings of eight or ten acres should be made about station houses to provide against danger from bush fires, if for no other purpose.

Very unstable swamps or muskegs are occasionally crossed by the method called "logging," which consists in placing logs transversely to the line under the bank, leaving their ends projecting at least six feet beyond the bottoms of the slopes. When this is done, the ground must not be broken for side ditches.

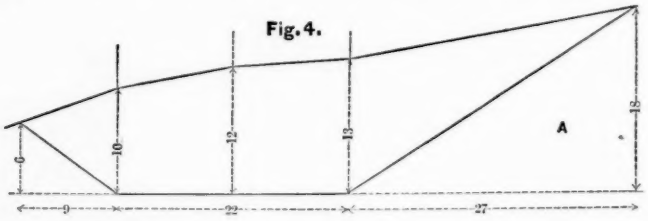
Sloping ground covered with pasture should be ploughed to prevent sliding, before the embankment be built thereon.

Measurements of material removed, should, whenever possible, be taken in excavation, and, when taken in embankment, due allowance should be made for both shrinkage and subsidence of the natural surface. Preliminary estimates are made from the profile by averaging end depths; quantities in ditches are taken out by the method of "end areas;" and in cuttings by either the last mentioned method, or by the "prismoidal formula," which is given by the equation $V = l \div 6(A + 4M + B)$, where V is the cubical content, l the length between the sections, A the area of one cross section, B that of the other, and M that of the cross section, which is not measured on the ground. Its value can be obtained by reducing each end area to an equivalent area between parallel bases by the formula $h = \frac{1}{3}(\sqrt{6A + b^2} - b)$, where b is the width of road bed and h the depth of the equivalent area, and calculating the area corresponding to the mean of the two depths thus found. It is sometimes obtained by calculating the area of an assumed cross section, all the variable dimensions of which are taken to be a mean between the corresponding dimen-

sions of the two end sections. The following example will serve to illustrate the different methods mentioned.

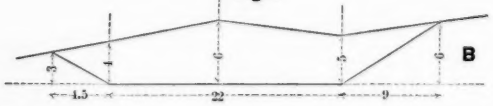
The easiest way to calculate the area of a cross section, whose dimensions are given, as in Figs. 4, 5 and 6, Plate I., is to add together twice the center depth

Fig. 4.



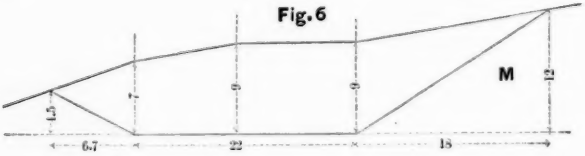
and the depth at each side of the road bed, multiply the sum by the width of the latter and divide by four. Then multiply each of the side depths by the horizontal distance of the slope stake from the side of the road bed, add the results together, divide by two and add

Fig. 5



to the quotient the area first found. By so doing there will result in this case, A=479 sq. ft, B=147 sq. ft., and M= 291.5 sq. ft. (from diagram). By the rule first given, M would equal 295.4 sq. ft., which shows how closely the two methods agree.

Fig. 6



If l=100,

By mean depth,....	V=1072	cubic yards.
By end areas,.....	V=1159	" "
By prismoidal formula, 1st case....	V=1116	" "
By prismoidal formula, 2d case....	V=1106	" "

The method of end areas always gives an excess, and the method of mean depths a deficiency. An average of the two in this case gives 1116.5 cubic yards, which agrees almost exactly with that obtained by the prismoidal formula—this merely accidental, owing, perhaps, to the similarity of the sections. On uneven ground cross sections ought not to be taken far enough apart to make the area of one more than double that of the next. In Henck's Field Book can be found formula for many special cases of earth measurement.

The office work of a section engineer comprises the making of a working profile and plan, and the filling out of a section book, cross-section sheets and return sheets. On the profile the final gradients should be drawn in red and the surface of the ground in black. It should be completed by drawing black vertical lines at every 100 feet, on which should be written in black figures the height of the surface of the ground above datum, as ascertained by the levels taken on the plugs; immediately above these should be placed the formation level in red figures; the height of fills should be placed below the surface heights, and the depth of cuts above formation level. Cuts and fills should be written in black figures having been as-

certained by subtracting surface and formation levels, the one from the other. At each change of gradient, the vertical lines should be in red and the height of formation level written in the same color, but with larger figures than at the intermediate stations. Where side ditches occur they should be indicated by blue lines at the level of their bottoms, and the center depth of cutting should be marked in blue figures on the black vertical lines. At the bottom of the profile should be shown, in blue, the curves in the line, with the stations where they begin and end, as well as their degrees of curvature; and at the top should be indicated the quality of the timber and ground, stream and road crossings, culverts, etc. In preparing plans, all *existing objects*, as roads, buildings, fences, etc., should be drawn in black lines; water should be indicated by a blue tint; and works *to be constructed*, as deviations of roads and streams, also the center line of the railway and the limits of land required for railway purposes should be shown in red lines.

The section book should be filled up so that it will form a complete and accurate record of all the longitudinal and transverse measurements and levels. In the upper portion of each page should be entered, in the proper place, the chainage to every station and intermediate point on the center line, each change of gradient, with its rate of inclination, the height of surface as well as formation level at each station and intermediate point, and the corresponding depth of cutting or filling. The lower part of the page should have every cross section regularly entered, giving the distances on each side of the center line, and the height above or depth below formation level; the slope stakes should also be designated. Tables of bench marks and reference hubs should be made at the beginning of the section book. Cross sections are plotted on paper ruled for the purpose into squares, each square representing one foot; their dimensions and areas should be marked thereon. The return sheets, which are large, should be ruled horizontally and vertically, and on them should be kept an account up to date of all material moved or used on the section. The vertical columns, which are of varying width according to what is to be written in them, should contain the following quantities (with others if need be), though not necessarily in the exact order here given: Stations from and to, clearing, close cutting, grubbing, line cutting, side ditches, borrow pits, off-takes, catch-water drains, rip-raps, masonry, stone drains, pole drains, piles, lumber, ties, timber and totals. The divisions, line cutting, side ditches, and off-takes should each be subdivided into three columns for earth, loose rock, and solid rock, borrow pits and catch-water drains into two each, for earth and loose rock; piles into two, for amount delivered and amount driven, and the timber into as many as there are different sizes used on the work. Grubbing, close cutting and clearing are measured by the acre; earth, loose rock, solid rock, rip-raps and masonry by the cubic yard; stone drains, pole drains, piles and timber by the linear foot; ties by number, and lumber by board measure. Before making out the first return sheet, the line should be divided into groups so that the cuts and fills will come in separate divisions. For instance, if the chainage of one end of a cutting were 1628+43 and the beginning of the next were 1640+64, the division, which would be for a fill, would be made from the first mentioned chainage to the last, and the quantities recorded would appear in the columns marked side ditches, off-takes and perhaps borrow pits. If the chainage at the end of second cutting were 1648+20, the division would include all between station 1640+64 and that station, and the quantities recorded would appear in the columns of line cutting and catch-water drains. When some of the divisions would thus be rendered too long for convenience in keeping the accounts, it will be necessary to subdivide them, taking, if practicable, the chainage of some natural or artificial feature of the ground, such as a stream, or a culvert opening as the point of division. The column for totals should be divided into three parts, for earth, loose rock, and solid rock. It is filled by adding up the quantities of each kind of material in each horizontal line, and entering the sum on that line in its proper place. After all the columns are filled, add them up vertically and place the sums so ob-

tained on a horizontal line below the columns, each in its proper place; then adding the total amounts of earth, loose rock and solid rock in this horizontal line (excluding those under the total columns), the amounts so obtained should equal those found by casting up the total columns. To find for what the contractor is to be paid, subtract from each of the totals the totals taken from the last return sheet. In this way can be readily seen by simply examining the return sheet, exactly what amount of work has been done and its distribution along the line.

Many of the directions here given for the manner of doing the office work are taken with very little change from the instructions given to the staff on the Canadian Pacific Railway; but as the officers of that road have devoted a great deal of attention to that subject, I think you will find their methods to be as good as those used on any of the railways on this side of the border.

After the grading has been completed and the ties have been distributed, the engineer re-locates the line on the grade very exactly, using a thin picket and driving thin stakes (lathes turned edgeways will do) at every hundred feet. He also drives very accurately grade plugs near the stations to the height of the rails.

To ballast the line the ties are laid and the rails spiked to them, so that the train can be run slowly over the road for the purpose of distributing the ballast. In unloading, the train must be kept working to and fro so as to thoroughly mix the different qualities of ballast, until a sufficient quantity is deposited for the first lift. The track must then be raised so that there will be an average depth of six inches beneath the sleepers, and the ballast must be well beaten and packed under and around them. As the raising proceeds the end of the lift should extend over not less than three rail lengths, and before trains are allowed to pass over the inclined portion of the track, it must be made sufficiently solid to prevent bending of the rails, or twisting the rail joints. A second lift is afterwards put on in the same manner. In wet cuttings an increased thickness of ballast is often necessary. The surface of ballast pits should be stripped of soil where such exists, and no material whatever should be placed on the road bed but good clean gravel, free from earth, clay, loam, or loamy sand; no large stones should be allowed, the maximum size of gravel being not greater than three inches in diameter.

A good deal more could be said on the subject of "construction," but as I mentioned in the beginning of these lectures, it is not my intention to write a complete treatise upon railroading, so I will refer you for further information to the works of Vose, Henck, Trautwine and Gillespie. However, as some of you may sometime go into contracting, I will close with a few remarks on that subject.

Railroad contracts are let sometimes by the mile, but more often by the cubic yard and other dimensions. To make a successful bid it is necessary to have a very good knowledge of the country through which the line is to pass, so as to be able to estimate the cost of plant, labor, etc. The usual form of tender indicates the probable amount of each kind of material, and the bids are based on these quantities. Now, if the person bidding feels positive that any of these quantities are in excess, and any others in reduction, he can, by bidding low on the former and high on the latter, keep down the sum total of his tender, and yet, if it be accepted, run a good chance of making a reasonable percentage. On large contracts it pays to send an expert over the line so as to ascertain pretty definitely how to bid. Care should be taken in purchasing supplies, plant etc., for the work; a great deal of money can be saved by knowing how and when to buy. Attention should be paid to the transportation of supplies, getting them in sufficient quantity when the rates are low, so as to tide over the time when they will have risen. It does not often pay to sub-let a cutting, owing to the expense of working it; if any one can make anything out of a cutting the contractor himself ought to be able to do it. But in regard to ditch work it is an entirely another affair. Men are unwilling, as a general rule, to undertake such disagreeable employment as ditching, unless they see a good chance of earning big wages. A grader will take out nearly twice the quantity of material per diem when working for himself, as

when he is paid by the day; so it is worth while for a contractor to sub-let swamp work to gangs of four or five, in which case it is well to make a written agreement with the men, binding them to do good work and to finish within a certain time. To insure the completion of the job, hold back ten per cent. of their pay until they finish everything satisfactorily. Never pay by bank measurement, for the navvies will be sure to fill up the bank with logs and stumps. A common trick of theirs, especially in winter when there is snow on the ground, is to pile about six inches of moss along the edges of the ditches so that when the measurements are made with a tape and rod, the depths will be in excess. In working wet cuttings it is often necessary to corduroy the road bed, both in the cutting itself and on the dump, to prevent the carts from sinking axle deep in the clay. If the engineer is a good-natured fellow, he will allow the ties to be used for the purpose. It does them very little harm, and saves a great deal of expense.

The best way to take out loose rock is by means of sledges. They can be made very cheaply by taking two of the spreading roots of a large tree and bracing them across; no artificial joint, unless it were an expensive one, could stand the stress that comes on the pointed end of the sledge. It is often the cheapest way to take out the gullet of the cut first: in which case it is advisable to allow a slope ratio of one horizontal to four vertical, so that if rock be struck there will be no difficulty in giving it the proper slope. Contractors sometimes find it to be economical to waste the materials from cuttings where the haul is a long one and the allowance for extra haul is insufficient to cover the extra expense. Loose rock is often hauled to the mouth of a cut and there wasted, the contractor replacing it at his own expense by an equal amount of earth. Rock cuts should be kept to the full width and depth as the work progresses, for it is very expensive taking up bottom. Ties should be cut and hauled in winter. It is to the mutual interest of contractor and engineer to keep liquor sellers away from the line.

In contracting, make your own measurements, not so much to keep a check on those of the engineer as to pay the workmen by, and to enable you to see the cost of all the different portions of work.

It is customary for contractors to keep a store for the purpose of supplying the workmen with everything they need for the work. If not carefully watched, they are liable to overrun their account, and, as they express it, to "jump the job."

When sub-letting a piece of work containing loose rock, it is better to bargain for a lump price than to pay so much for one kind of material and so much more for another, because an ignorant man is never satisfied with his measurements, in any case, and naturally counts upon a larger percentage of rock than he is likely to get.

No matter how scarce workmen may be, it is always good policy to discharge a mutinous man; it keeps the rest upon their good behavior. All the men on the work should be cautioned against removing any stakes, benches, or other landmarks of the engineer.

It is necessary, on a contract of any size, to keep plenty of plant on hand, especially such small articles as nails, bar iron, horseshoes, picks, shovels, irons for carts, etc. On this point I can speak from experience; for once, when in temporary charge of a five mile contract, I had to make a journey of one hundred and forty miles, half of the distance on foot through the swamp in order to purchase a few horseshoes, nails and axle irons; and had to have them packed thirty miles over the line. They were pretty expensive by the time they reached their destination. In contracting, as in everything else, it is better to do work well in the first place. One loses in reputation more than he saves in pocket by doing scrimp work, besides, in the end, he may have to expend far more for repairs than it would have cost in the first place to have completed everything according to the terms of the contract.

MM. Grimaux and Adam have recently accomplished the synthesis of citric acid from glycerine, and Kekulé announces that he has been working for the same object, but on different lines. He cannot tell at present whether he has been successful.

SANITARY ADMINISTRATION IN PARIS.

From "The Architect."

A REPORT has been presented to the Public Works Committee of Birmingham by Mr. Till, the Borough Surveyor, describing his visit to the principal Continental towns. He says that in Paris every possible opportunity was given to him for the inspection of the municipal works. The following are Mr. Till's notes on the water supply, gas, paving, and sewage of Paris:

Water Supply.—Until 1860 all water was supplied by the water company (Compagnie Générale des Eaux). In the year 1860 the City Council bought up all the company's rights and assets for an annuity, during fifty years, of £46,400, payable quarterly. At the same time the said company was constituted the administrative agent of the municipality for all matters relating to the future water supply. The company receives in this capacity a commission fixed thus: On all annual receipts between £144,000 and £240,000, 25 per cent.; between £240,000 and £400,000, 20 per cent.; between £400,000 and £480,000, 10 per cent.; on all over £480,000, 5 per cent. A long and careful inquiry by a special commission has just been completed, and their report presented. The main recommendation in the report is that private consumers shall have meters, and pay according to the quantity of water used. The present system of special contracts according to rent of house is found to be unsatisfactory and exceedingly wasteful. The water supplied to the city is of two qualities; the best comes from the Seine above the city, from the Vanne, the Dhuys, and from the wells, the supply from the latter being about 30 per cent. of the total daily supply. The Seine and other river water is not filtered before being delivered. The temperature of the well water is 39° F. The second quality is from the Marne and the canals. This is used for washing and watering the streets, and for industrial purposes generally, but not for cooking or drinking. It is intended to largely increase the supply of this quality of water, mainly for sanitary purposes. The municipality pays nothing for water used for public service.

Gas.—This does not belong to the city. It is the property of a company known as the "Company for Lighting and Heating by Gas." The charge for public lamps varies with the burner used. Of these there are three sizes, consuming respectively 100, 140, and 200 liters the hour. The charge to the city is, taking these in the same order, and per hundred hours, 1*s.* 3*d.*, 1*s.* 8*d.* and 2*s.* 6*d.* respectively. If supplied by meter the cost is 3*s.* 10½*d* per 1000 cubic feet. The private consumers may agree for so much per hour per burner, or may use an approved meter. In this case the maximum price permitted by law is 7*s.* 9*d.* per 1,000 cubic feet, being double what the city pays. The same applies to gas for heating or for gas engines. The illuminating power is fixed thus: Under a pressure of 3 millimeters, gas burning at the rate of 115 liters per hour shall give a light equal to the light of a Carcel lamp burning per hour 42 grammes of pure colza oil(3 millimeters, 0.12 inch). Where private consumers use meters they pay rent to the company. The city does not pay rent for meters registering gas used in lighting streets, but it does pay rent for those used in municipal buildings of all kinds. The gas company may demand monthly payment in advance. The company is not allowed to demand any other guarantee from the public. Recently a new mode of showing house numbers at night has been adopted. An upright frame in the form of a triangular prism, eight inches long, is fixed against the wall over the door. On each of the two visible sides is a plate of dark-blue glass, having the number in white letters. Inside the prism is a small gas jet. The first cost of this apparatus is £4, including fixing, and the cost of the gas about 22*s.* per annum.

Streets.—Of the whole city street area 73 per cent. is paved, 22 per cent. being macadamised, and 3 per cent. asphalted. It is intended to pave all streets where the traffic is heavy. The dimensions of pav-

ing stones have been the subject of careful inquiry and experiment, and the size finally adopted has a surface of 6¼ inches by 4 inches, and a depth of 6¼ inches. On steep gradients larger stones are used. Formerly the size was 9 inches each way. These stones were found to get rounded at the edges and corners, and to present an uneven surface. The steam rolling of the macadamised streets is done by contract, the cost being but little less than horse rolling. The asphalte carriage ways are bedded on beton 4 inches thick, well pressed down (in very wet weather bitumen is used instead, as the beton does not set quickly), over the beton a layer of mortar, and then, at the end of five days a layer of asphalt 1½ inch thick. The annual cost of footpaths of all kinds within the city is £43,600. The cleansing and flushing of streets and removal of house refuse costs annually £160,000. Each ward has its own set of sweepers, &c. Since 1873 a special tax has been levied for sweeping footpaths; previously each householder had to do that duty. There are in all 3,120 men and women employed in sweeping the streets and removing refuse. The street watering on paved streets is done from April 15 to September 30; on others from March 15 to October 15. About 22 per cent. of the area of the streets is watered by jointed pipes, and the rest by watering carts. The *first* mode is stated to cost only *about half as much*, area for area, as the *second* mode, but then the amount paid for horse hire is nearly double that in Birmingham. The watering carts belong to the municipality; the horses belong to contractors. The hydrants used for street watering are about 40 yards apart; the jointed pipes by which the water is distributed are 1 inch in diameter, in seven lengths of 8 feet each, on wheels, the joints being made with gutta-percha, the man with his finger on the nozzle regulating the quantity of water put on. We found in the Rue Castiglione, the carriageway whereof is 22½ yards wide, that it took the man, on an average, thirteen minutes to water only 33 yards in length of carriageway, and as each length is watered he uses a broom to sweep up the horse dung, &c. With one of our horses and carts a man can put on thirty-five loads, of 330 gallons each per day, and this quantity is sufficient to cover 82,000 square yards, or say 2,343 yards per cart load. I do not consider the hose system suitable for adoption here, except in the case of steep gradients like High Street and the Bull Ring, where a horse would work with difficulty. Since 1873 the house refuse has had to be placed each night in convenient boxes, in front of the houses, the contractor's carts calling for and removing the same early in the morning. The removal of this house refuse costs the city £80,000 per annum; formerly it was a source of income to the amount of £20,000 per annum. Two causes are alleged for the change: (1) deterioration in the value of the refuse, owing to the diminished use of *wood* as fuel, and increased use of coal and coke for houses and factories, and (2) increased distance of farms from the city, owing to the increase of suburban villas.

Trees in Streets and Squares.—Of these there are now about 90,000, besides 20,000 in the cemeteries. The roots of the trees planted in the streets and squares have a network of small drains for the supply of moisture. These are connected by a central drain, having a valve, and leading to the main sewer. The average cost of a tree, including transplanting, propping, draining, grid, &c., is £7, and about 90 per cent. of the transplanted trees succeed. There are three nurseries for trees, with an area of 6½ acres. The qualities sought in such trees are (*a*) rapid growth, (*b*) shade, (*c*) non-liability to attack by insects; and the trees selected on these grounds are chestnut, elm, the western plane, lime, and maple. 115 men are employed attending to the trees. The seats in streets and squares number 300.

Sewage.—We visited the great sewer in the Rue de la Pépinière, being met there by the officers of the municipality, detailed by order of M. Bufflers. This sewer was lighted up by means of fifty-six moderator lamps. The man in attendance showed the apparatus for pushing along sand and mud deposited on the bottom of the sewer. The sewer first inspected was of the second class. The water pipes and gas pipes are attached to the crown or sides of the sewers, thus avoiding tearing up the roadway when pipes need repairing. A truck, with paddle board moved by the

current, is used to force along the sand and mud. We next visited the main sewer, or collector of all sewage coming from the right bank of the Seine. The mean velocity in the collector was 165 feet per minute. The sewers receive (1) large portions of street sweepings; (2) storm water; (3) water thrown out from houses; (4) contents of water closets; (5) urine from closets fitted with double cylinder. From 500 to 600 men are employed in the sewers, of whom 32 work in the great collector. The length of this main sewer is 7½ miles, the length of main on the left bank of river is 5 miles, and the total length of sewers in the city is 500 miles. The daily outfall of all sewers is 300,000 cubic meters, or 66,000,000 gallons.

Cesspools.—Of these there are still in Paris about 240,000. In the new districts the double cylinder is used; the inner cylinder is pierced with holes to allow of the escape of liquid matter into the outer cylinder, and thence to the sewer. The solid matter is carried away in closed carts and dried at Bondy, and sold for manure. It retains but one-eighth of the nitrogenous elements, and is of little value as manure. The company that undertakes this work receives a yearly grant in aid.

Sewage Farms at Gennevilliers.—We visited the works at Asnieres, five miles below the city, being met by MM. Bufflers and Durand-Claye. A centrifugal pump, worked by a 400 horse-power Corliss engine, lifts in a day about 13,200,000 gallons of sewage (one-fifth of the total amount) to a height of 39 feet, into pipes which carry it across the Seine to the formerly arid chalky plain of Gennevilliers (a distance of about 6,600 feet) for distribution by means of open channels over the fields and gardens. The tube is 3 ft. 7 in. in diameter. The supply ceases when desired by the farmers. Besides this amount there are about 8,500,000 gallons daily received on another part of this plain from St. Ouen, from which it flows without pumping. The remainder of the city sewage is thrown direct into the Seine, causing a horrible stench for miles. The present pumping works have cost £120,000; the proposed further expenditure is £400,000. These works have been seven years in operation; the value of the land on the plain has increased on an average 450 per cent., and in one case 4,000 per cent., or a forty-fold increase. The plain contains 3,000 acres, of which 1,075 are treated with sewage. The main pipes are of beton, varying from 2 ft. to 4 ft. diameter; the smaller are of Doulton's glazed earthenware. Nine miles further from the city is a suitable tract of land, near St. Germains, with an area of 3,750 acres, and the city is in treaty with the State for its permanent acquisition. If successful, it is intended to turn all the sewage on to these two plains. We saw on the sewage farm all kinds of flowers, fruit, vegetables and trees suitable to the climate; the crops appeared healthy and abundant; the water that had passed through the soil seemed quite clean, and was tasteless. The drain pipes for collecting this filtered water are 6,600 ft. in length, the upper half, 3,300 ft., having holes; the lower part next the outfall being quite closed. The results of experiment so far, in yield, per acre, are: Artichokes, 20,000 to 30,000 heads; califlowers, 12,000 heads, weighing 16 tons; carrots, 32 to 52 tons; celery, over 40 tons; cabbage, 56 tons; onions, 24 to 32 tons; potatoes, 12 to 16 tons; pumpkins, 48 to 56 tons. About 10,000 cabbages are planted to the acre, and of these about two-thirds succeed. More than a million heads of cabbages are taken from the plain every year, each weighing on the average 11 lbs., or in all nearly 5,000 tons. It was stated that there was no bad taste or smell in the plants so long as only the roots were in contact with the sewage, and that the plants best suited to sewage treatment are green succulent ones. Young trees thrive remarkably well. The experiment has shown that trees well nourished when young thrive best ultimately, even if removed to a poorer soil. To trees the sewage supply must be stopped at the end of the summer. Mint and other plants, are cultivated for perfumes and for flavoring, and succeed remarkably well. Experiments extended over nine years show that on such soil as the plain of Gennevilliers, about 3,960,000 per acre per annum is the quantity that can be most effectively utilized. At this rate about 5,500 acres would suffice for the sewage of the city with its 2,000,000 of people. The sewage is now given gratis,

but it is proposed to make a small charge to prevent abuses. A Commission of Inquiry has just reported, and its conclusions may be summarized: (1) Green plants, cabbage, celery, lettuce, root crops, and "industrial plants," such as mint, are those best adapted to sewage treatment. (2) That the distribution of the sewage should be by open furrows, as at Gennevilliers. (3) That the supply should be intermittent and frequent. (4) That sewage be not allowed to touch any part of the plant except the root. (5) That the place of the furrows should be frequently changed.

THE RELATION BETWEEN ELECTRICITY AND LIGHT.*

From "Nature."

EVER since the subject on which I have the honor to speak to you to-night has arranged, I have been astonished at my own audacity in proposing to deal in the course of sixty minutes with a subject so gigantic and so profound, that a course of sixty lectures would be quite inadequate for its thorough and exhaustive treatment.

I must, indeed, confine myself carefully to some few of the typical and most salient points in the relation between electricity and light, and I must economize time by plunging at once into the middle of the matter without further preliminaries.

Now, when a person is setting off to discuss the relation between electricity and light, it is very natural and very proper to pull him up short with the two questions: What do you mean by electricity? and, What do you mean by light? These two questions I intend to try briefly to answer. And here let me observe that in answering these fundamental questions I do not necessarily assume a fundamental ignorance on your part of these two agents, but rather the contrary; and must beg you to remember that if I repeat well-known and simple experiments before you, it is for the purpose of directing attention to their real meaning and significance, not to their obvious and superficial characteristics; in the same way that I might repeat the exceedingly familiar experiment of dropping a stone to the earth if we were going to define what we meant by gravitation.

Now, then, we will ask first, what is electricity? and the simple answer must be, we don't know. Well, but this need not, necessarily, be depressing. If the same question were asked about matter, or about energy, we should have likewise to reply, no one knows.

But then the term matter is a very general one, and so is the term energy. They are heads, in fact, under which we classify more special phenomena.

Thus, if we were asked what is sulphur, or what is selenium, we should at least be able to reply, a form of matter; and then proceed to describe its properties, *i. e.*, how it affected our bodies and other bodies.

Again, to the question, what is heat? we can reply, a form of energy; and proceed to describe the peculiarities which distinguish it from other forms of energy.

But to the question, what is electricity? we have no answer pat like this. We cannot assert that it is a form of matter, neither can we deny it; on the other hand we certainly cannot assert that it is a form of energy, and I should be disposed to deny it; it may be that electricity is an entity *per se*, just as matter is an entity *per se*.

Nevertheless, I can tell you what I mean by electricity by appealing to its known behavior.

Here is a battery, that is, an electricity pump; it will drive electricity along. Prof. Ayrton is going, I am afraid, to tell you, on the 20th of January next, that it *produces* electricity; but if he does, I hope you will remember that that is exactly what neither it nor anything else can do. It is as impossible to generate electricity in the sense I am trying to give the word, as it is to produce matter. Of course I need hardly say that

* A lecture by Dr. O. J. Lodge, delivered at the London Institution, on December 16, 1880.

Prof. Aryton knows this perfectly well; it is merely a question of words, *i. e.*, of what you understand by the word electricity.

I want you, then, to regard this battery and all electrical machines and batteries as kinds of electricity pumps, which drive the electricity along through the wire very much as a water pump can drive water along pipes. While this is going on the wire manifests a whole series of properties, which are called the properties of the current.

[Here were shown an ignited platinum wire, the electric arc between to carbons, an electric machine spark, an induction coil spark, and a vacuum tube glow. Also a large nail was magnetized by being wrapped in the current, and two helices were suspended and seen to direct and attract each other.]

To make a magnet, then, we only need a current of electricity flowing round and round in a whirl. A vortex or whirlpool of electricity is, in fact, a magnet; and *vice versa*. And these whirls have the power of directing and attracting other previously existing whirls according to certain laws, called the laws of magnetism. And, moreover, they have the power of exciting fresh whirls in neighboring conductors, and of repelling them according to the laws of diamagnetism. The theory of the actions is known; though the nature of the whirls, as of the simple stream of electricity, is at present unknown.

[Here was shown a large electro magnet and an induction-coil vacuum discharge spinning round and round when placed in its field.]

So much for what happens when electricity is made to travel along conductors, *i. e.*, when it travels along like a stream of water in a pipe, or spins round and round like a whirlpool.

But there is another set of phenomena, usually regarded as distinct, and of another order, but which are not so distinct as they appear, which manifest themselves when you join the pump to a piece of glass or any non-conductor, and try to force the electricity through that. You succeed in driving some through, but the flow is no longer like that of water in an open pipe; it is as if the pipe were completely obstructed by a number of elastic partitions or diaphragms. The water cannot move without straining and bending these diaphragms, and if you allow it, these strained partitions will recover themselves and drive the water back again. [Here was explained the process of charging a Leyden jar.] The essential thing to remember is that we may have electrical energy in two forms, the static and the kinetic; and it is, therefore, also possible to have the rapid alternation from one of these forms to the other, called vibration.

Now we will pass to the second question: What do you mean by light? And the first and obvious answer is, everybody knows. And everybody that is not blind does know to a certain extent. We have a special sense-organ for appreciating light, whereas we have none for electricity. Nevertheless, we must admit that we really know very little about the intimate nature of light—very little more than about electricity. But we do know this, that light is a form of energy; and, morever, that it is energy rapidly alternating between the static and the kinetic forms—that it is, in fact, a special kind of energy of vibration. We are absolutely certain that light is a periodic disturbance in some medium, periodic both in space and time; that is to say, the same appearances regularly recur at certain equal intervals of distance at the same time, and also present themselves at equal intervals of time at the same place; that in fact it belongs to the class of motions called by mathematicians undulatory or wave motions. The wave motion in this model (Powell's wave apparatus) results from the simple up-and-down motion popularly associated with the term *wave*. But when a mathematician calls a thing a wave he means that the disturbance is represented by a certain general type of formula, not that it is an up-and-down motion, or that it looks at all like those things on the top of the sea. The motion of the surface of the sea falls within that formula, and hence is a special variety of wave motion, and the term wave has acquired in popular use this signification and nothing else. So that when one speaks ordinarily of a wave or undulatory motion one immediately thinks of something heaving up and down, or even perhaps of something breaking on the shore. But

when we assert that the form of energy called light is *undulatory*, we by no means intend to assert that anything whatever is moving up and down, or that the motion, if we could see it, would be anything at all like what we are accustomed to in the ocean. The kind of motion is unknown; we are not even sure that there is anything like motion in the ordinary sense of the word at all.

Now, how much connection between electricity and light have we perceived in this glance into their natures? Not much, truly. It amounts to about this: That on the one hand electrical energy may exist in either of two forms—the static form, when insulators are electrically strained by having had electricity driven partially through them (as in the Leyden jar), which strain is a form of energy because of the tendency to discharge and do work; and the kinetic form, where electricity is moving bodily along through conductors or whirling round and round inside them, which motion of electricity is a form of energy, because the conductors and whirls can attract or repel each other and thereby do work.

And, on the other hand, that light is the rapid alternation of energy from one of these forms to the other—the static form where the medium is strained, to the kinetic form when it moves. It is just conceivable, then, that the static form of the energy of light is *electro*-static, that is, that the medium is *electrically* strained, and that the kinetic form of the energy of light is *electro*-kinetic, that is, that the motion is not ordinary motion, but electrical motion—in fact that light is an electrical vibration, not a material one.

On November 5th, last year, there died at Cambridge a man in the full vigor of his faculties—such faculties as do not appear many times in a century—whose chief work has been the establishment of this very fact, the discovery of the link connecting light and electricity; and the proof—for I believe it amounts to a proof—that they are different manifestations of one and the same class of phenomena—that light is, in fact, an electro-magnetic disturbance. The premature death of James Clerk Maxwell is a loss to science, which appears at present, utterly irreparable, for he was engaged in researches that no other man can hope as yet adequately to grasp and follow out: but, fortunately, did not occur till he had published his book on "Electricity and Magnetism," one of those immortal productions which exalt one's idea of the mind of man, and which has been mentioned by competent critics in the same breath has the "Principia" itself.

But it is not perfect like the "Principia;" much of it is rough hewn, and requires to be thoroughly worked out. It contains numerous misprints and errata, and part of the second volume is so difficult as to be almost unintelligible. Some, in fact, consist of notes written for private use, and not intended for publication. It seems next to impossible now to mature a work silently for twenty or thirty years, as was done by Newton two and a half centuries ago. But a second edition was preparing, and much might have been improved in form if life had been spared to the illustrious author.

The main proof of the electro magnetic theory of light is this. The rate at which light travels has been measured many times, and is pretty well known. The rate at which an electromagnetic wave disturbance would travel if such could be generated (and Mr. Fitzgerald of Dublin thinks he has proved that it cannot be generated directly by any known electrical means) can be also determined by calculation from electrical measurements. The two velocities agree exactly. This is the great physical constant known as the ratio V, which so many physicists have been measuring, and are likely to be measuring for some time to come.

Many and brilliant as were Maxwell's discoveries, not only in electricity, but also in the theory of the nature of gases, and in molecular science generally, I can not help thinking that if one of them is more striking and more full of future significance than the rest, it is the one I have just mentioned—the theory that light is an electrical phenomenon.

The first glimpse of this splendid generalization was caught in 1845, five-and-thirty years ago, by that prince of pure experimentalists, Michael Faraday. His reasons for suspecting some connection between electricity and light are not

clear to us—in fact, they could not have been clear to him; but he seems to have felt a conviction that if he only tried long enough, and sent all kinds of rays of light in all possible directions across electric and magnetic fields in all sorts of media, he must ultimately hit upon something. Well, this is very nearly what he did. With a sublime patience and perseverance which remind one of the way Kepler hunted down guess after guess in a different field of research, Faraday combined electricity, or magnetism, and light in all manner of ways, and at last he was rewarded with a result. And a most out-of-the-way result it seemed. First, you have got to get a most powerful magnet and very strongly excite it; then you have to pierce its two poles with holes, in order that a beam of light may travel from one to the other along the lines of force; then, as ordinary light is no good, you must get a beam of plane polarized light and send it between the poles. But still no result is obtained until, finally, you interpose a piece of a rare and out-of-the-way material which Faraday had himself discovered and made, a kind of glass which contains borate of lead, and which is very heavy, or dense, and which must be perfectly annealed.

And now, when all these arrangements are completed, what is seen is simply this, that if an analyzer is arranged to stop the light and make the field quite dark before the magnet is excited, then—directly the battery is connected and the magnet called into action—a faint and barely perceptible brightening of the field occurs; which will disappear if the analyzer be slightly rotated. [The experiment was then shown.] Now, no wonder that no one understood this result. Faraday himself did not understand it at all; he seems to have thought that the magnetic lines of force were rendered luminous, or that the light was magnetized—in fact, he was in a fog, and had no idea of its real significance. Nor had any one. Continental philosophers experienced some difficulty and several failures before they were able to repeat the experiment. It was, in fact, discovered too soon, and before the scientific world was ready to receive it, and it was reserved for Sir William Thomson briefly, but very clearly, to point out, and for Clerk Maxwell more fully to develop, its most important consequences. [The principle of the experiment was then illustrated by the aid of a mechanical model.]

This is the fundamental experiment on which Clerk Maxwell's theory of light is based; but of late years many fresh facts and relations between electricity and light have been discovered, and at the present time they are tumbling in in great numbers.

It was found by Faraday that many other transparent media besides heavy glass would show the phenomenon if placed between the poles, only in a less degree; and the very important observation that air itself exhibits the same phenomenon, though to an exceedingly small extent, has just been made by Kundt Röntgen in Germany.

Dr. Kerr, of Glasgow, has extended the result to opaque bodies, and has shown that if light be passed through magnetized *iron* its plane is rotated. The film of iron must be exceedingly thin, because of its opacity, and hence, though the intrinsic rotating power of iron is undoubtedly very great, the observed rotation is exceedingly small and difficult to observe; and it is only by very remarkable patience and care and ingenuity that Dr. Kerr has obtained his result. Mr. Fitzgerald, of Dublin, has examined the question mathematically, and has shown that Maxwell's theory would have enabled Dr. Kerr's result to be predicted.

Another requirement of the theory is, that bodies which are transparent to light must be insulators or non-conductors of electricity, and that conductors of electricity are necessarily opaque to light. Simple observation amply confirms this; metals are the best conductors, and are the most opaque bodies known. Insulators, such as glass and crystals, are transparent whenever they are sufficiently homogeneous, and the very remarkable researches of Prof. Graham Bell in the last few months have shown that even *ebonite*, one of the most opaque insulators to ordinary vision, is certainly transparent to some kinds of radiation, and transparent to no small degree.

[The reason why transparent bodies must insulate, and why conductors must

be opaque, was here illustrated by mechanical models.]

A further consequence of the theory is that the velocity of light in a transparent medium will be affected by its electrical strain constant; in other words, that its refractive index will bear some close but not yet quite ascertained relation to its specific inductive capacity. Experiment has partially confirmed this, but the confirmation is as yet very incomplete. But there are a number of results not predieted by theory, and whose connection with the theory is not clearly made out. We have the fact that light falling on the platinum electrode of a voltameter generates a current, first observed, I think, by Sir W. R. Grove—at any rate it is mentioned in his "Correlation of Forces"—extended by Becquerel and Robert Sabine to other substances, and now being extended to fluorescent and other bodies by Prof. Minchin. And finally—for I must be brief—we have the remarkable action of light on selenium. This fact was discovered accidentally by an assistant in the laboratory of Mr. Willoughby Smith, who noticed that a piece of selenium conducted electricity very much better when light was falling upon it than when it was in the dark. The light of a candle is sufficient, and instantaneously brings down the resistance to something like one-fifth of its original value.

I could show you these effects, but there is not much to see; it is an intensely interesting phenomenon, but its external manifestation is not striking—any more than Faraday's heavy glass experiment was.

This is the phenomenon which, as you know, has been utilized by Prof. Graham Bell in that most ingenious and striking invention, the photophone. By the kindness of Prof. Silvanus Thompson I have a few slides to show the principle of the invention, and Mr. Shelford Bidwell has been good enough to lend me his home-made photophone, which answers exceedingly well for short distances.

I have now trespassed long enough upon your patience, but I must just allude to what may very likely be the next striking popular discovery, and that is the transmission of light by electricity; I mean the transmission of such things as views and pictures by means of the electric wire. It has not yet been done, but it seems already theoretically possible, and it may very soon be practically possible.

SCIENCE TEACHING IN THE PUBLIC SCHOOLS.

From the Proceedings of the American Association for the Advancement of Science.*

REPORT OF COMMITTEE ON SCIENCE TEACHING IN THE PUBLIC SCHOOLS:

THE Committee appointed at the Saratoga meeting of the American Association on Science Teaching in the Public Schools, respectfully submit the preliminary Report.

The repeated appointment by this body, in successive years, of committees to look into the scientific education of the public schools, must be taken as showing that such an inquiry is regarded as both legitimate and important. Yet the duties of such a committee have not been defined by the Association, nor have any of our predecessors opened the way to a consideration of the subject. It was probably expected that we would furnish a digest of information from many quarters, as to what sciences are taught in the public schools, with what facilities, and to what extent; accompanied by such recommendations regarding the increase of scientific studies as the results might suggest. But our course has not proved to be so clear. We have been arrested at the outset by a question of the quality of the science teaching in these schools which demands the first consideration. There are certain radical deficiencies in current science teaching, the nature and extent of which must be understood before any measures of practical improvement can be intelligently taken up. We shall here confine ourselves to this preliminary inquiry.

* The report of the Committee on Science Teaching in the Public Schools. The Committee consisted of E. L. Youmans, A. R. Grote, J. W. Powell, N. S. Shaler and J. S. Newbery.

The investigations has interest from the immense extent, and rapidly increasing influence, of the American public schools. There are now nearly one hundred and fifty thousand of these schools, supported at an annual expense of probably seventy or eighty million dollars. Maintained by state authority, they are firmly established in the respect and confidence of the community. Under the influence of normal schools, teachers' institutes, systematic superintendence, school boards, regulative legislation, and an extensive literature devoted specially to education, they have become organ ized into a system which is gradually growing settled and unified in its methods. With unbounded means and unlimited authority, these schools have undertaken to form the mental habits of the great mass of the youth of this country. They prescribe the subjects of study, the modes of study, and the extent and duration of studies for all the pupils that come under their charge. The sphere of their operations is, moreover, steadily extending. They are everywhere encroaching upon the province of higher education, everywhere trenching upon private schools and diminishing the interest in home education.

It may be assumed that the time has fully come when this system must be measured by the standards of science, and approved or condemned by the degree of its conformity to what these standards require. Science has become in modern times the great agency of human amelioration, the triumphs of which are seen on every hand and felt in all experience. Grave subjects are brought successively under its renovating and reconstructive influence; and latest and most important among them is the subject of education. Our inquiry now is, how far the public-school system has availed itself of the valuable aid that science offers in the proper cultivation of the minds of the young.

The interest and necessity of such an investigation will hardly be denied; but there may be a query as to its relevancy to the appropriate work of this society. The making of science popular was not among the objects for which our association was formed. Not that its founders were unmindful of the importance of widely diffusing the results of research; but they recognized that the interests of science are so vast, as to be only efficiently promoted by division of labor. Under the operation of this principle it was made the distinctive purpose of the association to contribute to the extension of original science by the discovery of new scientific truth, leaving its dissemination to the schools, the press, and the various agencies of public enlightenment. Nor does your committee understand that it is now proposed to depart from this policy; for the inquiry before us is really most pertinent to our special objects. It certainly cannot be a matter of indifference to this body, from its own point of view, how science is dealt with in the great system of schools which has undertaken the task of moulding the youthful mind of the country. We aim to advance science by the promotion of original investigation, which depends upon men prepared for the work. Do the schools of the nation, by their modes of scientific study, favor or hinder this object? Do they foster the early mental tendencies that lead to original thought; or do they thwart and repress them? We have an undoubted concern in this matter, and it is, moreover, strictly identical with that of the community at large; for there can be no better test than this of the real character of a school system. When we ask whether a mode of teaching and a manner of study are calculated to awaken the spirit of inquiry, to cultivate the habit of investigation, and rouse independent thought, our question goes to the root of all true education.

All sciences are the products of a method of thinking, and it is that method which concerns us when we propose to regard it as a means of mental cultivation. Science is an outgrowth of common knowledge, and the scientific method is but a development of the ordinary processes of thought that are employed by everybody. The common knowledge of people is imperfect because their observations are vague and loose, their reasoning hasty and careless, their minds warped by prejudice and deadened by credulity, and because they find it easier to invent fanciful explanations of things than to discover the real ones. For thousands of years the knowledge of nature

was rude and stationary because the habits of thought were so defective. But with a growing desire to understand how the world around is constituted, men improved their processes of thinking. They began, and were compelled to begin, by questioning accepted facts, and doubting current theories. The first step was one of self-assertion, implying that degree of mental independence which led men to think for themselves. They learned to make their own observations and to trust them against authority. It was found, as a first and indispensable condition of gaining clear ideas, that the mind must be occupied directly with the subject to be investigated. In this way scientific inquiry at length grew into a method of forming judgments which was characterized by the most vigilant and disciplined precautions against error. Of the mental processes involved in research it is unnecessary here to speak; we are only concerned to know that the scientific method is simply a systematic exercise in truth-seeking, and is the only mode of using the human mind when it is desired to attain the most accurate and perfect form of knowledge. The whole body of modern scientific truth, disclosing the order of nature and guiding the development of civilization, must be taken as an attestation of the validity of the scientific method of thought by which these results have been established. We here get rid of all cramping limitations. The scientific method is applicable to all subjects whatever that involve constancy of relations, causes and effects, and conform to the operation of law. It is applicable wherever evidence is to be weighed, error got rid of, facts determined, and principles established. Our public schools, unhappily, make but little use of this method in the work of mental cultivation, and we shall find some explanation of this by referring to the way they grew up.

The American public school system originated in the theory that the State owes to every child the rudiments of a common education, or an elementary knowledge of reading, writing and arithmetic, as implements of after mental improvement. But it was early found difficult to separate the primary use of tools from the acquisition of knowledge. Mr. Everett said: "I will thank any person to show why it is expedient and beneficial in the community to make public provision for teaching the elements of learning, and not expedient or beneficial to make similar provision to aid the learner's progress toward the mastery of the most difficult branches of science and the choicest refinements of literature." Under the influence of such considerations the rudimentary studies rapidly developed into sources of study embracing a variety of subjects. This led to the systematizing of instruction and the grading of schools, so that in nearly all the towns of the United States the public schools have been divided into primaries for the younger pupils and the grammar schools for older pupils; while within twenty-five years a third grade has arisen known as the high schools for the most advaneed students. In each division there are subgrades, and wherever improvements in public-school education are attempted, the principle of gradation is fundamental. So essential is it considered, that no aid is granted from the Peabody fund except to graded schools. As regards the plan of studies adopted there was no guiding principle. All sorts of subjects, and these for all sorts of reasons were taken up, and among them the sciences which are now regular parts of public school duty. Classes are found in physics, chemistry, mineralogy, geology, physiology, botany and zoology. There are text-books upon all these branches, graded to the varying capacities of learners. Teachers prepare in them, and in many cases apparatus is provided, and there are lectures with experiments, spe-specimens, maps and charts for illustrations.

The old ideal of a school is a place where knowledge is got from books by the help of teachers, and our public school system grew up in conformity with this ideal. The early effect of grading was to fix and consolidate imperfect methods. The sciences were assimilated to the old practice, and the science teaching falls short at just the points where it was inevitable that it should fall short. The methods of school teaching, and the habits of the teachers, had grown rigid under the *regime* of book-studies. As a consequence the science teaching in the public schools is

generally carried on by *instruction*. Through books and teachers the pupil is filled up with information in regard to science. Its facts and principles are explained as far as possible, and then left in the memory with his other school acquisitions. He learns the sciences much as he learns geography and history. Only in a few exceptional schools is he put to any direct mental work upon the subject-matter of science, or taught to think for himself.

As thus treated the sciences have but value in education. They fall below other studies as means of mental cultivation. Arithmetic arouses mental reaction. The rational study of language, by analytical and constructive tasks and the mastery of principles, strengthens the mental processes; but the sciences are not employed to train the faculties in the various ways to which they are severally adapted. They are not made the means of cultivating the observing powers, stimulating inquiry, exercising the judgment in weighing evidence, nor of forming original and independent habits of thought. The pupil does not know the subjects he professes to study by actual acquaintance with the facts, and he therefore becomes a mere passive accumulator of second-hand statements. But it is the first requirement of the scientific method, alike in education and in research, that the mind shall exercise its activity directly upon the subject matter of study. Otherwise scientific knowledge is an illusion and a cheat. As science is commonly pursued in book descriptions the learners cannot even identify the things they read about. As remarked by Agassiz, "the pupil studies Nature in the school-room, and when he goes out of doors he cannot find her." This mode of teaching science, which is by no means confined to the public schools, has been condemned in the most unsparing manner by all eminent scientific men as a "deception," a "fraud," an "outrage upon the minds of the young," and "an imposture in education."

Nor has this criticism of bad practices been without its effect. We are met by the statement that much has been done in the public schools to escape the evils of mere book science. The method of object lessons has been extensively introduced into primary schools with the professed purpose of cultivating the powers of observation in childhood. It is claimed that this is a beginning in science; and, as it brings the mind into action upon things, is a corrective of the inordinate study of words. But object teaching has not yielded what was expected of it; and is in no true sense a first step in science. Nothing is gained educationally by barely having an object in hand when it is talked about. Myriads of objects are present to the senses of people but no insight follows. The observing faculties must be tasked if they are to be trained. The pupil is not to have the properties of objects pointed out, but he is to find them out. Science will do its work of educating the observing faculties only as they are quickened and sharpened by exercise in discrimination. The scientific aim is to replace vague confused impressions by clear and accurate ideas. Skill in the detection of nice distinctions is only gained by prolonged and careful practice. Object lessons afford no such cultivation. We do not say they are useless, but they are not the A B C of science, and do not, as a matter of fact, open the way to the proper study of the special sciences. This is their test and their condemnation. When the primary pupils have gone over their prescribed course of object lessons, and are passed on to a higher grade, strange to say the "objects" are suddenly dropped as if the objective method had been exhausted. In the technical phrase *perceptive* education is to be replaced by *conceptive* education. Instruction in elementary science is now to be carried on by what is known as oral teaching. This method as extensively practised in the grammar grades of the public schools is everywhere growing in favor, and we are once more told that it is a successful revolt against book studies. It is chiefly applicable to the sciences and its cardinal idea is instruction without a text book. This looks fair but it is delusive. The method does not remove the book that the pupil may come at the phenomena, but it removes the book that the teacher may take its place. Oral teaching is class instruction, in which information is imparted in a familiar manner with the view of awakening the interest of the class. But so far as real science is concerned it is doubtful if this method

is not worse than the one it replaces. Following the maxim of certain German educators that "the teacher is the school," it was assumed that when apathy prevails in the school room it is solely the teacher's fault. Oral exercises enable them to escape this reproach by giving animation to school work. It is said that this is a "live system" in contrast to the old humdrum routine of lessons and recitations. But science gets no real help. There is only the substitution of a superficial class activity for the more deliberate work of the individual pupil. More mental effort is required on his part to get a lesson from a book than to listen to a lesson given by the teacher. The teacher is to do everything, and stands in the place not only of the book but of the pupil also. Is this not a step backward in education? The teacher is magnified at the expense of close study, and science is cheapened by the method. Oral teaching implies a fertility, a versatility, and a proficiency in scientific knowledge on the part of teachers which that class of persons does not possess. It is a premium on tutorial smattering and cramming by which the voluble teacher with superficial acquisitions and a ready memory becomes the model teacher. There may be benefits in this method, but science does not gain them. Judicious oral assistance, as in the physical, chemical, or natural history laboratory, given by a competent master to a pupil at work, is invaluable for stimulus and guidance; but the aid must be discreet and the skillful teacher will not talk too much. But where it is all talk and no work, and text books are filtered through the very imperfect medium of the ordinary teacher's mind, and the pupil has nothing to do but to be instructed, every sound principle of education is outraged, and science is only made ridiculous.

This failure to gain the benefits of real scientific study has its source deep in the constitution of the public schools. In dealing with masses of children, classification became necessary which gave rise, as we have seen, to grading and an elaborate mechanical system. The working of children in lots seems to be a necessity of the public schools, but it strengthens the practice of verbal instruction recitations and lesson giving. It is well fitted to impress the public with the idea that there is much done in the schools. There is a prescribed routine of operations and a display of order that is admired. But teacher and learner are subordinated to the system. It is machine work and machines make no allowances. Gradation assumes and enforces a uniformity among pupils which is not according to the facts. Wide personal differences of capacity, aptitude, attainment and opportunity, not only exist among children, but they are the prime data of all efficient mental cultivation. In the graded schools, just in proportion to the perfection of the mechanical arrangements, individuality disappears. Special original capacity, the main thing, counts for nothing. The mind cannot be trained in such circumstances to originate its own judgments. The exercise of original mental power or independent inquiry is the very essence of the scientific method and with this the practice of the public schools is at war. Moreover, a system which deals with the average mind and does not get at the individual mind breaks down at the point where all true education really begins, that is, in promoting self-culture. The value of educational systems consists simply in what they do to incite the pupil to help himself. Mechanical school work can give instruction, but it cannot develop faculty, because this depends upon self-exertion. Science, if rightly pursued, is the most valuable school of self-instruction. From the beginning men of science have been self-independent and self-reliant because self-taught; and it is a question whether they have been most hindered or helped by the schools. De Candolle, in his valuable book on the conditions which favor the production of scientific men, says that the discoverers, the masters of scientific method have chiefly appeared in small towns where educational resources have been scanty; and that they have often been most helped by the very poorness of their teaching which threw them back upon themselves. It was to their advantage that the schools were not so perfect as to extinguish individuality and thus destroy originality.

Our strictures are here upon the general working of the public school system; but we recognize that there are many exceptional teachers who do what

they can to deal with science in the true spirit, while multitudes of instructors are chafing under present restrictions and groping after something better. The bad system is moreover continued chiefly from the lack of knowledge as to the possibilities of a better. But the better method of teaching science has been proved entirely practicable. The institution where we meet and many other science schools have shown it. A large number of teachers have demonstrated that various branches of science can be taught to the young by the true as well as by the false method. What is now most urgently needed is to gather from these experiences practical plans of improvement in science teaching for the benefit of those who desire better guidance than they now have.

In his address as Rector of the University of Aberdeen, Professor Huxley said: "I would not raise a finger to introduce more book work into every art curriculum in the country." We concur in this view as applied to the present science teaching in our public schools. We would not raise a finger to extend it. President Barnard, of Columbia College, in a public address reprobating in severe terms the common method of teaching science as being an inversion of the true order of cultivating the mental faculties, referred to the great benefits which must arise "when our systems of education shall have been remodeled from top to bottom." That result may come about in the fulness of time, but it is wise to expect only a slow and gradual improvement. Vice-President Grote, in his St. Louis address, pointed out the guiding principle in this case as a substitution of real knowledge for second-hand information by a necessary law of mental advancement. In obedience to this principle, the cultivators of original science should do what they may to raise the standard of our prevalent science teaching; and we respectfully ask that the Association will assign to a committee the duty of reporting at our next meeting on the best modes of improving the teaching of science in our public schools.

ON DUST, FOGS, AND CLOUDS.*

By JOHN AITKEN.

From "Nature."

Dust, fogs, and clouds seem to have but little connection with each other, and we might think they could be better treated of under two separate and distinct heads. Yet I think we shall presently see that they are more closely related than might at first sight appear, and that dust is the germ of which fogs and clouds are the developed phenomena.

This was illustrated by an experiment in which steam was mixed with air in two large glass receivers; the one receiver was filled with common air, the other with air which had been carefully passed through a cotton-wool filter and all the dust removed from it. In the unfiltered air the steam gave the usual and well-known cloudy form of condensation, while in the filtered air no cloudiness whatever appeared. The air remained supersaturated and perfectly transparent.

The difference in the behavior of the steam in these two cases was explained by corresponding phenomena, in freezing, melting, and boiling. It was shown that particles of water vapor do not combine with each other to form a cloud particle, but the vapor must have some solid or liquid body on which to condense. Vapor in pure air therefore remains uncondensed or super-saturated, while dust particles in ordinary air form the nuclei on which the vapor condenses and forms fog or cloud particles.

This represents an extremely dusty condition of the air, as every fog and cloud particle was formerly represented by a dust particle, which vapor by condensing upon it has made visible. When there is much dust in the air but little vapor condenses on each particle,

* Abstract of a paper read to the Royal Society of Edinburgh.

and they become but little heavier, and easily float in the air. If there are few dust specks each gets more vapor, is heavier, and falls more quickly.

These experiments were repeated with an air-pump, a little water being placed in the receiver to saturate the air. The air was then cooled by slightly reducing the pressure. When this is done with unfiltered air a dense cloudiness fills the receiver, but when with pure air no fogging whatever takes place, there being no nuclei on which the condensation can take place. In this experiment, and in the one with steam, the number of cloud particles is always in proportion to the dust present. When the air is nearly pure and only a few dust particles present, then only a few cloud particles form, and they are heavy and fall like fine rain.

The conclusions drawn from these experiments are: (1) that whenever water vapor condenses in the atmosphere it always does so on some solid nucleus; (2) that dust particles in the air form the nuclei on which the vapor condenses; (3) that if there was no dust there would be no fogs, no clouds, no mists, and probably no rain, and that the supersaturated air would convert every object on the surface of the earth into a condenser on which it would deposit; (4) our breath when it becomes visible on a cold morning, and every puff of steam as it escapes into the air, show the impure and dusty condition of our atmosphere.

The source of the fine atmospheric dust was then referred to, and it was shown that anything that broke up matter into minute parts would contribute a share. The spray from the ocean, when dried and converted into fine dust, was shown to be an important source. Meteoric matter also probably contributed a proportion. Attention was then directed to the power of heat and combustion as a source of this fine dust.

It was shown that if there is much dust then each particle only gets a little vapor condensed upon it, that when the particles are numerous they become but little heavier, and easily float in the air, and give rise to that close packed but light form of condensation which constitutes a fog, and therefore whatever increases the amount of dust in the air tends to increase fogs, and that when the dust particles are not so numerous the cloud particles are larger and settle down more quickly.

It was shown that by simply heating any substance, such as a piece of glass, iron, brass, &c., a cloud of dust was driven off, which, when carried along with pure air into the experimental receiver, gave rise to a dense fog when mixed with steam. So delicate is this test for dust that if we heat the one-hundredth of a grain of iron wire the dust driven off from it will give a distinct cloudiness in the experimental receiver, and if we take the wire out of the apparatus and so much as touch it with our fingers and again replace it, it will again be active as a cloud producer. Many different substances were tried, and all were found to be active fog producers. Common salt is perhaps one of the most active.

Heat, it is well known, destroys the motes in the air, and it might be thought that flame and other forms of combustion ought to give rise to a purer air. Such, however, is not the case. Gas was burned in a glass receiver, and supplied with filtered air for combustion, and it was found that the products of combustion of pure air and dustless gas gave rise to an intensely fog-producing atmosphere. It may be mentioned here that the fog-producing air from the heated glass, metals, and burning gas were each passed through the cotton-wool filter, and the air was in all cases made pure, and did not give rise to cloudiness when mixed with steam.

It will be seen that it is not the dust motes which are revealed to us by a beam of sunlight when shining into a darkened room, that form the nuclei of fog and cloud particles, as these may be entirely removed by heat, and yet the air remain active as a cloud producer. The heat would seem to break up the larger motes which reflect the light into smaller and invisible ones. When speaking of dust, it is to these infinitesimally small and invisible particles we refer. The larger motes which reflect the light will no doubt be active nuclei, but their number is too small to have any important effect.

It is suggested, and certain reasons are given for supposing, that the blue

color of the sky is due to this fine dust.

Other experiments were made to test the fog-producing power of the air and gases from different sources. The air to be tested was introduced into the experimental receiver and mixed with steam, and the relative densities of the fog produced were noted. It was always found that the air of the laboratory where gas was burning gave a denser fog than the air outside, and that the air outside varied, giving less fog during wet than during dry weather. The products of combustion of gas burned in a Bunsen flame, a bright flame, and a smoky flame, were all tested and found to be about equally bad, and all much worse than the air in which they were burned. Products of combustion from a clear fire and from a smoky one gave about equal fogging, and both much worse than the air of the room.

Experiments were made by burning different substances. Common salt when burned in a fire or in alcohol flame gave an intensely fog-producing atmosphere, but burned sulphur was the most active substance experimented on. It gave rise to a fog so dense it was impossible to see through a thickness of 5 cm. of it.

The vapors of other substances than water were tested to see if they would condense in the cloud form without nuclei on which to deposit. All the substances experimented on, which included sulphuric acid, alcohol, benzole, and paraffin, only gave a cloudy condensation when mixed with ordinary unfiltered air, and remained perfectly clear when mixed with filtered air, all these acting like water vapor.

Before referring to fogs, which have now become so frequent and aggravated in our large towns, it was pointed out that caution was necessary in applying the results of the experiments.

The conditions of a laboratory experiment are so different, and on so small a scale, that it is not safe to carry their teaching to the utmost limits and apply them to the processes which go on in nature. We may, however, look to the experiments for facts from which to reason, and for processes which will enable us to understand the grander workings of nature.

It having been shown that vapor, by condensing on the dust particles in the air, gives rise to a fogging, the density of which depends on the amount of fine dust in the air; the more dust the finer are the fog particles, and the longer they remain suspended in the air. It having been also shown that all forms of combustion, perfect and imperfect, are producers of fog nuclei, it is concluded that it is hopeless to expect that, adopting more perfect forms of combustion than those at present in use, we shall thereby diminish the frequency, persistency, or density of our town fogs. More perfect combustion will, however, remove the pea-soup character from the fogs and make them purer and whiter, by preventing the smoke which at present mixes with our town fogs and aggravates their character, and prevents them dissolving when they enter our rooms. Smoke descends during a fog, because the smoke particles are good radiators, and soon get cooled and form nuclei on which the water vapor condenses. The smoke thus becomes heavier and falls. This explains why falling smoke is often a sign of coming rain. It indicates a saturated condition of the atmosphere.

Sulphur when burned has been shown to be an intensely active fog producer. Calculation shows that there are more than 200 tons of sulphur burned with the coal every winter day in London, a quantity so enormous as quite to account for the density of the London fogs. It is suggested that some restriction ought to be put on the amount of sulphur in the coal used in towns.

Before utterly condemning the smoke and the sulphur, it was pointed out that it would be necessary thoroughly to investigate and fully to consider the value of smoke as a deodorizer, and also the powerful antiseptic properties of the sulphurous acid formed by the burning sulphur. The air during fogs is still and stagnant. There is no current to clear away the foul smells and deadly germs that float in the air, which might be more deadly than they are, were it not for the suspended soot and burned sulphur. We must therefore be on our guard lest we substitute a great and hidden danger for an evident but less evil.

ON THE CAUSE OF CAVITIES IN CAST STEEL.

By DR. F. C. G. MULLER, of Brandenburg.

Translated from Glaser's Annalen, for Foreign Abstracts of Institution of Civil Engineers.

The author refers to his researches on the Bessemer process, and on the gas bubbles in Bessemer steel, published in the "Zeitscrift des Vereines deutscher Ingenieure," 1878 and 1879.* The present paper is designed to put his results in a more concise and practical form, and to reply to objections made by certain French metallurgists, especially M. Pourcel.

In the ordinary process of casting steel ingots by running the hot metal into cold ingot moulds, much difficulty is found in obtaining thoroughly sound castings. This arises mainly from two evils, due to entirely different causes; (1) the central funnel-shaped cavity, due to shrinkage of the interior, after the outside is set; (2) dispersed cavities, due to gases imprisoned in the cooling metal. The former always makes the top of the ingot worthless, and sometimes extends far into the interior; but this can be prevented by proper attention in casting, and by special methods. The latter is much more difficult of cure. The fluid metal always holds a considerable quantity of occluded gas, and this, when the metal sets, draws together into small bubbles. In many kinds of steel the ingot boils up during the process of running. Such ingots are to be looked on as sponges filled with gases, and if cut across when cold, are found to present innumerable worm-shaped cavities. If, as is usually done, the boiling is stopped by covering in the ingot with sand, the bubbles are found to be arranged radially in two rows all round the outside of the section, while the central part, and also a skin about half an inch thick, is quite sound. The reason of this phenomenon is that the metal, in splashing down into the mould, loses a part of its occluded gas, and therefore is no longer in a saturated condition. When, therefore, the outside begins to set, the gases first draw into the interior until this becomes saturated, whereupon the formation of bubbles begins, and the mass boils over. The stopping of the mould with sand next causes a severe pressure within the still fluid parts, and the gases are thus kept in solution (or prevented from forming bubbles) even during the process of setting. This pressure is itself a source of danger, as, if the sand is removed too soon an explosion may take place.

These defects are not peculiar to Bessemer steel. They are also found in Siemens-Martin and crucible steel, but intermittently; appearing and disappearing without visible cause. Even if an ingot does not boil, and has an apparently sound head, it may still have cavities here and there in its bulk. It is true that with rolled or forged steel these cavities do not prevent perfectly sound work being produced, though they necessitate greater care, and extra hammering or squeezing, in the subsequent processes. Even rails with honeycombed skins (where the thin outer layer has given way in the furnace, and allowed the cavities to communicate with the air) may give thoroughly good results when tested. But for steel castings such cavities are obviously prejudicial, and for many purposes have altogether prevented their use.

The exact nature of the gases thus occluded has been much discussed, it being generally supposed that they are mainly carbonic oxide. Gautier was the first to treat the point as being settled by experience, and he announced that the soundness of the castings in Terre Noire steel, and also in Krupp's and in the Bochum steel, was due to the presence of silicon, neutralizing the carbonic oxide. This was generally accepted, in spite of the known fact that most of the porous German and Austrian steels were quite as rich in silicon as those above mentioned. The matter was in this condition when the author resolved to test it by direct

* Vide Minutes of Proceedings Inst. C.E., vol. lvi. p. 360; and vol. lx., p. 494.

experiment.* The experiments showed that the gas consisted, not of carbonic oxide, but of hydrogen mixed with a little nitrogen. This was verified not only for cast steel, but for many different descriptions of iron. Confining the inquiry to steel, two cylinders of Bochum steel, made from the same perfectly sound ingot, but one forged and the other not gave the following results:

	Percentage of gas.	Composition.			
		H.	Ni.	C.	Tot.
Unforged steel	15.5	92.4	5.9	1.4	99.7
Forged steel†	5.5	73.4	25.3	1.3	100.0

This proves that a certain quantity of gas is occluded among the molecules of the steel, even where there are no visible cavities, and that this gas consists of hydrogen, with about one-fifth of nitrogen. The latter is no doubt derived from the air, and the hydrogen from the moisture contained in the large quantity of air which mixes with the metal, especially in the Bessemer process. The absence of CO is remarkable, because it has been shown by Troost and Hautefeuille that it is readily absorbed by red hot iron, and because it is developed in large quantities in the converter. But with fluid iron it either is not absorbed at all, or in small proportions compared with hydrogen and nitrogen. It may, however, act in another way, namely, by freeing the metal from dissolved gases, in the same way as when water is saturated with an easily soluble gas, and then exposed to a current of gas which will not dissolve in the water, the latter carries away with it a large portion of the former gas. This may explain why samples taken in the middle of the Bessemer process are often quite sound, while those taken towards the end, when the development of CO has ceased, are porous. It also explains the fact that at the Bochum and Hosch (Dortmund) works, absolutely sound Bessemer ingots are obtained by blowing the charge completely dead, and then pouring in 7 per cent. of fluid spiegeleisen. A violent reaction takes place, due to the carbon of the speigeleisen combining with the oxygen of the metal to form CO, and this probably carries off mechanically the hydrogen dissolved in the metal, which would otherwise form cavities in the cold ingot. This was proved by analyzing the gases which escaped, at Bochum, in this final reaction, and which in two cases had the following composition:

	CO.	CO_2	N.	H.	O.	Total.
Analysis No. 1.	82.6	—	14.3	2.8	—	99.70
" No. 2.	78.55	0.86	16.38	2.52	1.32	99.63

It will be seen that hydrogen was present, and in about the proportion demanded by the theory; so that in fact CO is not only innocent of the production of cavities in steel, but may actually be used to prevent them.

Another mode of attaining this end would be by thoroughly drying the air before blowing it through the metals, so as to stop the introduction of hydrogen. This could be done by using burnt lime, with little expense. The nitrogen would, of course, remain, but it is uncertain whether this, by itself, would be harmful.

These discoveries of the author were subsequently criticised by M. Pourcel, who suggested that the hydrogen was due to the ingot having been quenched from a red heat in water; that the water had penetrated into the cavities and decomposed, the oxygen going to oxidize the walls, while the hydrogen remained. This criticism, however, falls to the ground, because the ingots were not quenched as suggested.* In fact this could not have been done without making the steel too hard to bore. Moreover, although red hot steel will decompose steam, there is no reason to suppose that it will decompose cold water; *e.g.*, there is no evidence that hydrogen exists in quenched iron. Nor is it easy to see how hydrogen so formed could have found its way into the interior cavities of the ingot, and certainly not under a pressure of seven atmospheres. It would seem as if the French metallurgists were thinking only of the central cavity, which is open to the atmosphere, and which, of course, was not under consideration at all.

* For the process and results, see Minutes of Proceedings Inst. C.E., vol. lvi., p. 360; and vol. lx., p. 494.

† Mr. E. Windsor Richards has obtained a very similar analysis to this for the gases in steel ingots (Proc. Inst. Mech. Eng. 1880, p. 402).

* In his preliminary report to the German Chemical Society, Dr. Muller mentions that the ingots were cooled in water; it is to be understood, therefore, that this cooling was not begun till their temperature had fallen too low to admit of hardening.

THE GEOMETRICAL INTERPRETATION OF IMAGINARY QUANTITIES.

Translated from the French of M. Argand by Prof. A. S. HARDY.

Contributed to VAN NOSTRAND'S ENGINEERING MAGAZINE.

IV.

The treatise, by Argand, appeared in the year 1806.* In Vol. IV., 1813–14, of Gergonne's *Annales de Mathematiques*, appeared an article entitled "New Principles of the Geometry of Position and Geometrical Interpretation of Imaginary Symbols," by J. F. Francais, Professor in the Imperial School of Artillery at Metz, of which the following is an abstract.

The author began by calling attention to the distinction between the magnitude and position of a line, and to the still incomplete state of the geometry of position. He proposed the notation a_α, b_β, ... to represent right lines whose absolute lengths were a, b, the subscript Greek letters denoting the angles made by these lines with any arbitrary axis of reference. Francais used the expression "lines given in magnitude and position," to designate what Argand called "directed right lines." In the term *ratio* he included the relative position as well as the relative magnitude, four directed lines being in proportion as

$$a_\alpha : b_\beta :: c_\gamma : d_\delta,$$

when $\frac{b}{a}=\frac{d}{c}$, and also $\beta-\alpha=\delta-\gamma$. In such a proportion, the absolute lengths are in geometrical, while the angles made with the axis are in arithmetical progression; and the homologous sides of any two similar complanar figures are in proportion. In conformity with the above definition, the proportion

$$a_\alpha : b_\beta :: b_\beta : c_\gamma$$

involves the equations

$$\frac{b}{a}=\frac{c}{b}, \text{ and } \beta-\alpha=\gamma-\beta,$$

whence $\beta=\frac{1}{2}(\alpha+\gamma)$,

or a mean proportional between the directed lines bisects their included angle. So for the continued proportion

$$a_\alpha : b_\beta : c_\gamma \ldots . : l_\lambda : m_\mu,$$

we should have

$$\frac{b}{a}=\frac{c}{b}=\ldots . =\frac{m}{l},$$

and $\beta-\alpha=\gamma-\beta=\mu-\lambda$.

He then proposed a second notation. By the former $a_0=a$, and $\mathbf{1}_0=1$; therefore $\mathbf{1} : \mathbf{1}_\alpha :: a : a_\alpha$, or $a_\alpha=a.\mathbf{1}_\alpha$; so that a directed line might also be represented by the symbol $a.\mathbf{1}_\alpha$, a denoting its length and $\mathbf{1}_\alpha$ its position.

Lines parallel to the axis of reference drawn from left (right) to right (left) were distinguished as positive (negative); angles estimated above (below) the axis from right to left were regarded positive (negative). This convention, in connection with the above notation, gave $+1=1_0$, $-1=1_{\pm\pi}$, and therefore

$$+a=a\times(+1)=a.1_0,$$
$$\text{and } -a=a\times(-1)=a.1_{\pm\pi}.$$

And from the known relations

$$+1=e^{0\pi\sqrt{-1}}, \text{ and } -1=e^{\pm\pi\sqrt{-1}},$$

results

$$+a=a\times(+1)=a.e^{0\pi\sqrt{-1}},$$
$$\text{and } -a=a\times(-1)=a.e^{\pm\pi\sqrt{-1}}.$$

He then proceeded to establish four theorems:

I. *In the geometry of position, imaginary quantities of the form* $\pm a\sqrt{-1}$ *represent perpendiculars to the axis of reference, and, conversely, perpendiculars to the axis are imaginaries of this form.*

Demonstration.—The quantity $\pm a\sqrt{-1}$ is a mean proportional between $+a$ and $-a$, that is, between a_0 and a_π; hence by the definition of mean proportional is expressed by $a_{\pm\frac{\pi}{2}}$; or, it is perpendicular to the axis and drawn either above or below it; and we have

$$+a\sqrt{-1}=a_{+\frac{\pi}{2}}, \text{ and } -a\sqrt{-1}=a_{-\frac{\pi}{2}}.$$

Reciprocally, every perpendicular to this axis is represented, in conformity with the above

* See January, February and March Magazine.

notation, by $a_{\pm\frac{\pi}{2}}$ and is, therefore, by definition, a mean proportional between a_0 and $a_{\pm\pi}$, or between $+a$ and $-a$. It is therefore an imaginary of the form $\pm a\sqrt{-1}$.

Cor. 1.—As signs of position, $\pm\sqrt{-1}$ is identical with $1_{\pm\frac{\pi}{2}}$.

Cor. 2.—Moreover, since $-1=1_{\pm\pi}=e^{\pm\pi\sqrt{-1}}$, we have also $\pm\sqrt{-1}=1_{\pm\frac{\pi}{2}}=e^{\pm\frac{\pi}{2}\sqrt{-1}}$.

Cor. 3.—So-called imaginary quantities are quite as real as positive or negative quantities, and differ from them only in position, being in fact perpendicular to them.

M. Francais argued that this theory of signs was more consistent than the ordinary one of Cartesian geometry, where, as abscissas and ordinates, two kinds of positive and two kinds of negative quantities were admitted. He contended that having once defined positive and negative quantities, as laid off parallel to the axis of abscissas, it was illogical to admit others not comprised in the definition, and that the common theory was thus faulty in admitting two incompatible principles where one was sufficient.

THEOREM II.—The sign of position $1_a=e^{a\sqrt{-1}}$.

Demonstration.—Let the semi-circumference of a unit circle be divided in the direction of positive arcs into m equal parts, and radii be drawn to the points of division; these radii will form a progression both as to magnitude and position, by definition. The two extremes being $1_0=+1$, and $1_\pi=-1=e^{\pi\sqrt{-1}}$, the means $1_{\frac{\pi}{m}}, 1_{\frac{2\pi}{m}}, \ldots\ldots 1_{\frac{(m-1)\pi}{m}}$, will be

$$e^{\frac{\pi}{m}\sqrt{-1}}, e^{\frac{2\pi}{m}\sqrt{-1}}, \ldots\ldots e^{\frac{(m-1)\pi}{m}\sqrt{-1}};$$

or, in general, $1_{\frac{n\pi}{m}}=e^{\frac{n\pi}{m}\sqrt{-1}}$; and as $\frac{n\pi}{m}$ may be any angle whatever, we have finally $1_a=e^{a\sqrt{-1}}$.

From this theorem M. Francais drew the following corollaries:

1. That by taking the logarithms of each member of the last equation, $a\sqrt{-1}=\log(1_a)$; showing that, in the geometry of position, arcs of circles are the logarithms of the corresponding radii, being affected with the sign $\sqrt{-1}$ since they are perpendicular to the axis of reference; explaining also the expression, "imaginary arcs of a circle are logarithms," and giving a rational interpretation of the symbolic equation

$$\frac{\pi}{2}\sqrt{-1}=\log(\sqrt{-1}).$$

2. That since $a_a=a.1_a$, we have also $a_a=a.e^{a\sqrt{-1}}$.

3. That since $e^{a\sqrt{-1}}=\cos a+\sin a\sqrt{-1}$, it follows that $a_a=a\cos a+a\sin a.\sqrt{-1}$, or that to express a directed right line, we must take the sum of its projections in two rectangular co-ordinate axes; each projection being taken with its proper sign of position.

4. That for any such lines we may substitute any number, provided that the sum of the projections of the latter is equal to the sum of the lines themselves: that is, we may write $a_a, b_\beta, \ldots. m_\mu$ for x_ξ, provided we have

$$\text{(A)}\quad x.e^{\xi\sqrt{-1}}=a.e^{a\sqrt{-1}}+b.e^{\beta\sqrt{-1}}+\ldots.+m.e^{\mu\sqrt{-1}},$$

$$\text{or (B)}\quad \begin{cases} x\cos\xi=a\cos a+b\cos\beta \\ \qquad\qquad +\ldots.+m\cos\mu, \\ x\sin\xi=a\sin a+b\sin\beta \\ \qquad\qquad +\ldots.+m\sin\mu, \end{cases}$$

and conversely.

If the lines a_a, b_β, x_ξ, etc. form a closed polygon, (B) will be satisfied, and hence for any given line may be substituted a series of others, forming with it a closed polygon; conversely for a series of lines forming an unclosed polygon may be substituted the closing line.

The application of these remarks to the theory of the composition and resolution of forces is evident. On this point M. Francais briefly says, "This theory which has always involved some difficulties is thus reduced to a problem of the Geometry of Position."

THEOREM III.—The sign of position 1_a may also be written $1^{\frac{a}{2\pi}}$, that is to say $1_a=1^{\frac{a}{2\pi}}$.

Demonstration.—If the unit circle be divided into m equal parts and the radii be drawn, they will form a progression whose extremes are unity. Hence $1_{\frac{2\pi}{m}}=1^{\frac{1}{m}}$, $1_{\frac{4\pi}{m}}=1^{\frac{2}{m}}$, ... $1_{\frac{2n\pi}{m}}=1^{\frac{n}{m}}$. Let then $\frac{2n\pi}{m}=a$; we shall have $\frac{n}{m}=\frac{a}{2\pi}$, and consequently $1_a=1^{\frac{a}{2\pi}}$.

Cor. 1.—It follows from this theorem: 1°, that the above radii denote the m mth roots of unity; 2°, these roots are all equal, differing only in their positions; 3°, they are all equally

real, being represented by lines given both in magnitude and position.

Cor. 2.—Comparing the last two theorems we obtain at once the well-known values of these roots, which may be expressed, in general, by

$$1^{\frac{n}{m}} = e^{\frac{2n\pi\sqrt{-1}}{m}} = \cos\frac{2n\pi}{m} + \sin\frac{2n\pi}{m} \cdot \sqrt{-1}.$$

He then proposes the substitution, for $+$, $-$ and $\pm\sqrt{-1}$, of 1_0, $1_{\pm\pi}$, $1_{\frac{\pi}{2}}$, in connection with the general sign $1_{\pm a}$; an additional advantage over that already suggested being that $+$ and $-$ will indicate addition and subtraction only, and so have but one meaning.

THEOREM IV.—All the roots of an equation of any degree are *real* and may be represented by lines given in magnitude and position.

Demonstration.—It has been shown that every equation of any degree whatever is always decomposable into real factors of the first or second degree, and hence it is sufficient to show that the root of an equation of the second degree can be represented by lines given in magnitude and position. Now the roots of an equation of the second degree, being of the form $x = p \pm \sqrt{q}$, can at once be constructed by the foregoing rules; for, 1°, if q is positive, x will be the sum or difference of two positive or negative quantities, laid off on the axis; 2°, if q is negative, x will be a right line drawn from the origin, the co-ordinates of whose extremity are p and $\sqrt{q}$.

M. Francais concluded his communication as follows:

Such is a very brief sketch of the new principles on which it seems to me desirable and necessary to found the geometry of position, and which I submit to the judgment of geometers. Being in direct conflict with the commonly received ideas concerning so-called imaginary quantities, I expect they will encounter many objections; but I dare to think that their thorough examination will show them to be well founded, and that the consequences I have drawn from them, strange as at first sight they may seem, will nevertheless be found in harmony with the most rigorous logic. I ought, moreover, to acknowledge that the germ of these ideas is not my own. I found it in a letter to my late brother from M. Legendre, in which this great geometer gave (as something he himself had derived from another, and purely as a matter of curiosity) the substance of my definitions of proportion and ratio—theorem I. and cor. 3 of theorem II.; but the latter was a mere suggestion, and only justified by a few applications. To myself, therefore, belongs only the credit due to the manner in which these principles have been set forth, and their proof, the notation, and the proposal of the symbol $1 \pm a$. I hope that the publicity thus given to my own results may induce the real author of these conceptions to make himself known and to publish his own researches on this subject.

In the same volume (IV. p. 71–73) of the annales which contained this paper from M. Francais, a note was inserted by the editor, Gergonne, to the effect that two years before (1811) in a letter he had written to M. de Maiziére on a communication which the latter had contributed to the first volume, he had suggested that "numerical quantities were perhaps improperly classified in a single series, and that, from their very nature, it seemed as if they should be arranged in a table of double argument, as follows:

$$\begin{array}{c}
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\ldots, -2+2\sqrt{-1}, \ -1+2\sqrt{-1}, \\
+2\sqrt{-1}, +1+2\sqrt{-1}, +2+2\sqrt{-1}, \ldots \\
\ldots, -2+\sqrt{-1}, -1+\sqrt{-1}, +\sqrt{-1}, \\
+1+\sqrt{-1}, \ +2+\sqrt{-1}, \ldots \\
\ldots, -2, \quad -1, \quad \pm 0, \quad +1, \quad +2, \quad , \ldots \\
\ldots, -2-\sqrt{-1}, -1-\sqrt{-1}, -\sqrt{-1}, \\
+1-\sqrt{-1}, \ +2-\sqrt{-1}, \ldots \\
\ldots, -2-2\sqrt{-1}, -1-2\sqrt{-1}, -2\sqrt{-1}, \\
+1-2\sqrt{-1}, \ +2-2\sqrt{-1}, \ldots \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots

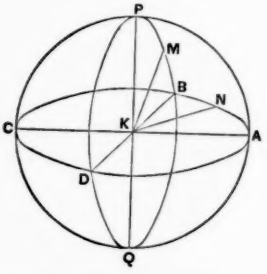
strange as would seem, since several had entertained them, and in closing he remarks that M. Francais' paper may be summarized in the following proposition:

"When, we seek a determinate but unknown length which is supposed to lie in a certain direction along a given line from a given point, while it really lies in the opposite direction, we obtain a negative expression; and if this length is not on the line at all, the expression will appear under an imaginary form."

M. Francais' paper called forth a second article from M. Argand, which appeared in Vol. IV., p. 133–147 of the Annales, wherein he called attention to his previous publication, and claims to have been the person to whom Legendre referred in his letter, he having submitted his first treatise to Legendre's examination. This second paper is, in the main, a restatement of the views advanced in the first; but in it he abandoned the use of the signs $\sim$ and $\dashv$, and returned to that of $\pm\sqrt{-1}$. He also added some further remarks, which are interesting as showing how he attempted to extend his theory to tri-dimensional space, and of which the following is a translation:

Let (Fig. 23) $\overline{KA}=+1$, $\overline{KC}=-1$, $\overline{KB}=+\sqrt{-1}$, $\overline{KD}=-\sqrt{-1}$; any other radius $\overline{KN}$, in the same plane, will be of the form $p+q\sqrt{-1}$; and, conversely, every expression of this form will denote a directed line of this plane.

Fig. 23



* Draw now from the center K a perpendicular KP=KA to the plane. How shall this directed line be designated? Is it wholly independent of $\overline{KA}$ and $\overline{KB}$, or can it be referred analytically to the prime unit $\overline{KA}$, as are $\overline{KB}$, $\overline{KC}$? Guided by analogy it would seem that, taking the entire circumference as the unit angle, a directed radius making an angle a with $\overline{KA}$ would be expressed, from the principles already laid down, by 1^a; but this expression would be troublesome when a is a fraction, because it would then have more than one value. This objection would be met by the adoption of M. Francais' notation, 1_a; we should thus have $\overline{KA}=1_0$, $\overline{KB}=1_{\frac{1}{2}}$, $\overline{KC}=1_1$, $\overline{KD}=1_{\frac{3}{2}}$. We have considered angles reckoned from A above and below as positive and negative. Now, if we apply to the angles the rule we have adopted for lines, we should be led to regard imaginary angles as laid off in a direction perpendicular to that which corresponds to real angles. Suppose the semi-circumference ABC to revolve about AC, the point B describing the circle BPDQ; since we already have

$$\text{angle } \overline{AKB}=+\tfrac{1}{2}=\tfrac{1}{2}.(+1),$$

$$\text{angle } \overline{AKD}=-\tfrac{1}{2}=\tfrac{1}{2}.(-1),$$

we may write angle $\overline{AKP}=\frac{1}{2}\sqrt{-1}=\frac{1}{2}.1_{\frac{1}{2}}$; whence we conclude that

$$\overline{KP}=1_{\frac{1}{2}}\ 1_{\frac{1}{2}}=1_{\frac{1}{2}}\sqrt{-1}=1^{\frac{1}{2}\sqrt{-1}}=$$

$$(1^{\frac{1}{2}})^{\sqrt{-1}}=(\sqrt{-1})^{\sqrt{-1}}.$$

This would seem to be the analytical expression required.

If on the circle BPD we take the point M, so that $\overline{BKM}=\mu$, we shall have, in like manner, angle $\overline{AKM}=\frac{1}{2}(\cos\mu+\sqrt{-1}\sin\mu)$, and writing for brevity $\cos\mu+\sqrt{-1}\sin\mu=\rho$, $\overline{KM}=1_{\frac{1}{2}\rho}=1.^{\frac{1}{2}\rho}=(1^{\frac{1}{2}})^{\rho}=(\sqrt{-1})^{\cos\mu+\sqrt{-1}\sin\mu}$ will be the general expression for all radii perpendicular to the primitive radius $\overline{KA}$.

Let us now seek an expression for $\overline{BKP}$. On the circumference ABC, the angles estimated from B in either direction are positive and negative, and real, and the plane BKP is perpendicular to their direction; it would thus seem that the angle $\overline{BKP}$, like $\overline{AKP}$, $=\frac{1}{2}\sqrt{-1}$, and that this should in like manner be true for any angle $\overline{NKP}$, N being on the circumference ABCD; but that this conclusion is erroneous is evident from the fact that when N and C coincide, we should have $\overline{CKP}=\frac{1}{2}\sqrt{-1}$, whereas this angle is evidently $-\overline{AKP}=-\frac{1}{2}\sqrt{-1}$. To avoid this difficulty, observe that having adopted a direction for $+1$, there are an infinity of lines perpendicular to it, among which one is arbitrarily chosen as that of $\sqrt{-1}$. The general expression for every unity taken in one of these directions is, as we have just seen,

$$1_{\frac{1}{2}\rho}=1^{\frac{1}{2}\rho}=(\sqrt{-1})^{\rho}=(\sqrt{-1})^{\cos\mu+\sqrt{-1}\sin\mu}.$$

Conceive at the point A, an infinite number of directions perpendicular to the circumference at that point; one of these will be that of $\overline{KP}$; namely that one we have taken to construct the positive imaginary angles $+a\sqrt{-1}$; that is, for this case we have taken $\rho=1=\overline{KA}$. So, at C, the direction parallel to $\overline{KP}$ gave negative imaginary angles $-a\sqrt{-1}$; that is, we have made $\rho=-1=\overline{KB}$. Hence, with respect to the direction from B parallel to $\overline{KP}$,

analogy would lead us to make $\rho = \sqrt{-1} = \overline{KB}$. Thus the expression for $\overline{BKP}$ will be $\frac{1}{2}(\sqrt{-1})^{\sqrt{-1}}$.

We will not further enlarge on these suggestions, and observe only in closing that the expressions a, a_b, a_{b_c}, which designate lines considered in reference to one, two and three dimensions, are only the first terms of a series which can be indefinitely extended.

If the above ideas are admissable, the question so often raised, as to whether every function can be reduced to the form $p+q\sqrt{-1}$, would be answered in the negative; and $\overline{KP} = (\sqrt{-1})^{(\sqrt{-1})}$ would offer the simplest example of a quantity irreducible to this form, and as heterogeneous with respect to $\sqrt{-1}$ as is the latter with respect to $+1$.

It is true there are demonstrations going to show that the form $(a+b\sqrt{-1})^{m+n\sqrt{-1}}$ can always be reduced to the form $p+q\sqrt{-1}$; but we may be permitted to remark that those which make use of series are not conclusive so long as it is not proved that p and q are finite. Indeed it often happens in analysis that a series, which, from its very nature can only be true for real quantities, assumes an infinite value, or rather form, when it is made to represent an imaginary quantity; and in like manner it is presumable that a series composed of terms of the form $p+q\sqrt{-1}$ or a_b can become infinite if it is to express a quantity of the order a_{b_c}. As for those demonstrations which employ logarithms, they also seem somewhat obscure, because we have as yet no definite conceptions of imaginary logarithms. It is also necessary to ascertain whether the same logarithm may not belong at the same time to several quantities of different orders; a, a_b, a_{b_c}. Moreover the several values resulting from the radicals of the proposed expression is another source of ambiguity, so that one may succeed in rigorously reducing $(a+b\sqrt{-1})^{m+n\sqrt{-1}}$ to the form $p+q\sqrt{-1}$ without its being necessarily true that this expression has no other values of the order a_{b_c} irreducible to this form.

Before this second paper of Argand's had come to the notice of Francais, the latter also had endeavored to extend the new theory of imaginaries to tri-dimensional space. In the fourth Vol. of Gergonne's Annales a letter appeared from Francais, from which the following is an extract:

According to my previous definition, positive and negative angles are taken in the same plane, which for brevity I shall designate as the plane xy. It would then seem natural to suppose that imaginary angles are situated in planes perpendicular to xy, and this supposition would be justified by analogy alone; but its legitimacy may be shown as follows: the angle $+\beta\sqrt{-1}$ is a mean proportional, both as to magnitude and position, between $+\beta$ and $-\beta$; it is therefore situated with respect to the angle $+\beta$ as is the angle $-\beta$ with respect to it, which can only be so long as the plane of the angle $\pm\beta\sqrt{-1}$ bisects the angle of the planes $+\beta$ and $-\beta$. Now these planes coincide; therefore the plane of $\pm\beta\sqrt{-1}$ is perpendicular to the plane xy. Conversely, since every plane perpendicular to xy bisects the angles between the planes of the positive and negative angles, every angle β, in such a plane, may be considered a mean proportional in magnitude and position between $+\beta$ and $-\beta$; hence its value, in respect to both magnitude and position, is $\pm\beta\sqrt{-1}$.

From the above, and my 2nd and 3d theorems, it follows that $1_{\beta\sqrt{-1}} = e^{(\beta\sqrt{-1})\sqrt{-1}} = e^{-\beta}$

$= 1^{\frac{\beta\sqrt{-1}}{2\pi}} = \cos(\beta\sqrt{-1}) + \sqrt{-1}\sin(\beta\sqrt{-1})$.

Lambert's *hyperbolic sine and cosine* are thus reduced to the theory of circular arcs, Naperian logarithms, and roots of unity.

It further follows that

$$1_a \cdot 1_{\beta\sqrt{-1}} = e^{a\sqrt{-1}} e^{(\beta\sqrt{-1})\sqrt{-1}} = e^{(a+\beta\sqrt{-1})\sqrt{-1}} = 1_{a+\beta\sqrt{-1}}$$

$$= e^{a\sqrt{-1}}[\cos(\beta\sqrt{-1}) + \sqrt{-1}\sin(\beta\sqrt{-1})]$$

$$= \cos a\cos(\beta\sqrt{-1}) + \sqrt{-1}\sin a\cos(\beta\sqrt{-1}) + \sqrt{-1}.e^{a\sqrt{-1}}\sin(\beta\sqrt{-1}).$$

Whence

$$a_{a+\beta\sqrt{-1}} = a\cos a\cos(\beta\sqrt{-1}) + \sqrt{-1}\, a\sin a\cos(\beta\sqrt{-1}) + \sqrt{-1}.ae^{a\sqrt{-1}}\sin(\beta\sqrt{-1}).$$

Hence the projections of a on the three coordinate axes, or rather its three components, will be

$$a\cos a\cos(\beta\sqrt{-1}),\ \sqrt{-1}.a\sin a\cos(\beta\sqrt{-1}),\ \sqrt{-1}.a_a\sin(\beta\sqrt{-1}).$$

These, Monsieur, are the results I have reached; but I confess I am not yet satisfied with them. I desire to suppress wholly the old imaginary notation, as I have done for geometry of two dimensions; that is, for the latter I have reduced oblique lines of the form $A+B\sqrt{-1}$ to that of a_a, where a denotes the absolute length of the line, and a the angle it makes with the axis of reference. In tri-dimensional geometry, I desire to express the position of any line by a_{a_A}, a denoting the absolute length a the above angle and A the angle made by the plane of a with xy; but as yet all my efforts in that direction have proved unsuccessful. I trust some one more skillful than myself may succeed in filling up this gap. At all events, I am confident that the true method of extending our theory of imaginaries to tri-dimensional geometry consists in the consideration of imaginary angles.

In a postscript to this letter, Francais acknowledges the receipt of Argand's memoir, and that to the latter belongs the credit of the discovery of the geometrical representation of imaginaries.

He then adds:

In starting from the same principle we have

reached different results. I have said above that I have not succeeded in reducing the expression for the position of any right line in space to the form a_{a_A}. The reasons for my failure are these: I attempted to make, from analogy

$$a_A = a.e^{A\sqrt{-1}} = a(\cos A + \sqrt{-1}\sin A)$$ whence

$$1_{a_A} = \left(e^{a\sqrt{-1}}\right)^{e^{A\sqrt{-1}}} =$$

$$(\cos a + \sqrt{-1}\sin a)^{\cos A + \sqrt{-1}\sin A},$$

which, when $a = \frac{1}{2}\pi$, $A = \frac{1}{2}\pi$, gives

$$1_{\frac{1}{2}\pi.1_{\frac{1}{2}\pi}} = (\sqrt{-1})^{\sqrt{-1}},$$

which agrees with the result of M. Argand. But, developing the general case, we have

$$1_{a_A} = \left(e^{a\sqrt{-1}}\right)^{e^{A\sqrt{-1}}} = e^{\left(a.e^{A\sqrt{-1}}\right)\sqrt{-1}} =$$

$$e^{(a\cos A + \sqrt{-1}\,a\sin A)\sqrt{-1}} =$$

$$e^{\sqrt{-1}.a\cos A}.e^{(\sqrt{-1}a\sin A)\sqrt{-1}}$$

$$= [\cos(a\cos A) + \sqrt{-1}\sin(a\cos A)] \times$$

$$[\cos(\sqrt{-1}.a\sin A) + \sqrt{-1}\sin(\sqrt{-1}.a\sin A)] =$$

$$\cos(a\cos A)\cos(\sqrt{-1}.a\sin A) + \sqrt{-1}$$

$$\sin(a\cos A)\cos(\sqrt{-1}.a\sin A) +$$

$$\sqrt{-1}.e^{\sqrt{-1}.a\cos A}.\sin(\sqrt{-1}.a\sin A),$$

an expression which, on account of its doubly transcendental character, would seem inadmissable. On comparing it with

$$1_{\lambda+\mu\sqrt{-1}} = \cos\lambda\cos(\mu\sqrt{-1}) + \sqrt{-1}\sin\lambda\cos$$

$$(\mu\sqrt{-1}) + \sqrt{-1}.e^{\lambda\sqrt{-1}}\sin(\mu\sqrt{-1})$$

I rejected it altogether, because the angles a and A are easily found in terms of μ and λ by spherical trigonometry. In fact we have

$$\cos\lambda\cos(\mu\sqrt{-1}) = \cos a,$$

$$\sin\lambda\cos(\mu\sqrt{-1}) = \sin a\cos(A\sqrt{-1}),$$

$$\sin(\mu\sqrt{-1}) = \sin a\sin(A\sqrt{-1});$$

whence

$$\cos\lambda = \frac{\cos a}{\sqrt{1-\sin^2 a\sin^2(A\sqrt{-1})}},$$

$$\sin\lambda = \frac{\sin a\cos(A\sqrt{-1})}{\sqrt{1-\sin^2 a\sin^2(A\sqrt{-1})}}.$$

And therefore

$$1_{a_A} = [\cos a + \sqrt{-1}\sin a\cos(A\sqrt{-1})] \times$$

$$\left\{1 + \frac{\sin a\sin(A\sqrt{-1})}{\sqrt{1-\sin^2 a\sin^2(A\sqrt{-1})}}\sqrt{-1}\right\}.$$

From this it seems to me clear that a_A cannot be determined as a_a was, and that the supposed analogy between angles and lines does not exist.

You must have remarked, Monsieur, that M. Argand does not prove my proposition $a_a = a(\cos a + \sqrt{-1}\sin a)$, and that this fundamental equality is, with him, simply a supposition justified only by a few examples.

On this remark M. Gergonne very justly observes that no demonstration was needed, inasmuch as Argand had *defined* the sum of directed lines as a certain composition of motions, "a very natural extension of the ordinary definition of Algebra."

M. Francais concludes:

I do not quite see why M. Argand (No. 12), in writing $2\pi = 1$, should introduce a new unit, rendering, it seems to me, the rest of his paper obscure. Finally, I should be loth to admit the correctness of his assertion that

$$(e^{\sqrt{-1}})^{d\sqrt{-1}}$$

is irreducible to the form $A + B\sqrt{-1}$. In fact, we have

$$e^{\sqrt{-1}} = e^{\log(e^{\sqrt{-1}})} = e^{\log e + \log\sqrt{-1}}$$

$$= e^{\log e + \frac{1}{2}\pi\sqrt{-1}} = .e^{\log e}\, e^{\frac{1}{2}\pi\sqrt{-1}};$$

therefore,

$$(e^{\sqrt{-1}})^{d\sqrt{-1}} = e^{(d\log e)\sqrt{-1}}e^{-\frac{1}{2}d\pi} = e^{-\frac{1}{2}d\pi}$$

$$[\cos(d\log e) + \sqrt{-1}\sin(d\log e)],$$

which is certainly of the form $A + B\sqrt{-1}$. I, therefore, think myself correct in regarding the expression $(e^{\sqrt{-1}})^{d\sqrt{-1}}$, which he assigns to the third dimension, as simply a conjecture open to serious objection.

On Nov. 13th, 1813, M. Servois addressed a letter to Gergonne, which is especially interesting as bearing upon the extension of Argand's theory to space of three dimensions. He objected first to Francais' proof of his first theorem. This proposition, that $\pm a\sqrt{-1}$ is a mean proportional in magnitude and position between $+a$ and $-a$, he claimed to consist of two, one of which, viz: that $\pm a\sqrt{-1}$, was a mean proportional as to position is not evident, and is indeed precisely what is to be proved. To this criticism Gergonne replied that, although Servois thinks it evident that $\pm a\sqrt{-1}$ is a mean as regards *magnitude*, between $+a$ and $-a$, it seemed to him difficult to see how such an expression, which, with its signs, is a *negation* of *magnitude*, could be a mean between two *reals*; that as regards magnitude, the mean could only be a; but, taking position into account, the mean must also be conceived under this new aspect, and is for this very reason a mean in position as well as magnitude, so that the interpretation of $\pm a\sqrt{-1}$ is reduced to the selection of a line which is situated with reference to $+a$ as $-a$ is to it.

Servois objected, secondly, that the new theory was not only founded merely on analogy, but was not even justified *a posteriori* by its applications. Emphasizing Argand's remark that it consisted in the use of a special notation, he char-

acterized it as "a sort of geometric mask, superadded to analytic forms whose direct use was more simple and expeditious." For example, he says:

. . . take Argand's first application, where he proposes to develope $\sin(a+b)$ and $\cos(a+b)$. From the general formula $e^{a\sqrt{-1}} = \cos a + \sqrt{-1}\sin a$, I obtain $e^{(a+b)\sqrt{-1}} = \cos(a+b) + \sqrt{-1}\sin(a+b)$, and thence $e^{(a+b)\sqrt{-1}} = e^{a\sqrt{-1}}\, e^{b\sqrt{-1}} = (\cos a + \sqrt{-1}\sin a)(\cos b + \sqrt{-1}\sin b)$, or $e^{(a+b)\sqrt{-1}} = (\cos a\cos b - \sin a\sin b) + \sqrt{-1}(\sin a\cos b + \cos a\sin b)$; equating these two values of $e^{(a+b)\sqrt{-1}}$, and subsequently the real and imaginary parts separately, we have $\cos(a+b) = \cos a\cos b - \sin a\sin b$, $\sin(a+b) = \sin a\cos b + \cos a\sin b$. All the other geometrical applications are easily made in the same manner. They may be found in various works, and especially in "A Purely Algebraic Theory of Imaginary Quantities," by M. Suremain-de-Missery (Paris, 1801). The single application to algebra (close of Argand's treatise) seems to me quite unsatisfactory. I do not think it sufficient to find values for x which render the polynomial of less and less value; it is necessary, besides this, that the law of decrease should necessarily render it *zero*; and that it should be such that zero is not, so to speak, the asymptote of the polynomial.

After citing Euler's proof that

$$(\sqrt{-1})^{\sqrt{-1}} = e^{-\frac{1}{2}\pi},$$

in reply to Argand's assertion that this expression was irreducible to the form $p+q\sqrt{-1}$, he raises two other objections, which are important and given in full.

Accustomed to designate the position of a point in a plane by an angle and radius vector, geometers have certainly not been ignorant of the consequences of M. Francais' definition. . . . . But, content with distinguishing between the magnitude and position of a right line in a plane, they had not yet formed, from these two simple ideas, a single complex one, or rather they had not yet created a new *etre geometrique*, uniting at once both the ideas of magnitude and position. The length of a right line and its position, *i. e.* the angle it makes with a fixed axis, are two quantities, which we may term homogeneous; now, how can they be so combined as to form this new entity called a directed line? It seems to me this problem is not yet satisfactorily solved. If a is the length of the line and α the arc of the unit circle which measures the angle it makes with a fixed axis, undoubtedly we may in general represent the line by $\phi(a,\alpha)$, and the function ϕ must be determined by the essential condition it is to satisfy. Thus 1°, evidently $\phi(a,\alpha) = +a$ must correspond to $\alpha = o$, $\alpha = 2\pi, \ldots$, $\alpha = 2n\pi$, and $\phi(a,\alpha) = -a$ to $\alpha = \pi$, $\alpha = 3\pi, \ldots$, $\alpha = (2n+1)\pi$; 2°, also, evidently from $\phi(a,\alpha) = \phi(b,\beta)$ we must have $a = b$, $\alpha = \beta$. But 3°, does it follow from the proportion $\frac{\phi(a,\alpha)}{\phi(b,\beta)} = \frac{\phi(c,\gamma)}{\phi(d,\delta)}$, as M. Francais says, that we must have $\frac{a}{b} = \frac{c}{d}$ and $\alpha - \beta = \gamma - \delta$? I do not see that this necessarily follows from the conception of ϕ. The very meaning of this ratio $\frac{\phi(a,\alpha)}{\phi(b,\beta)}$ is quite obscure. What indeed is meant by doubling, trebling, etc, a directed line? *A priori* this is not intelligible. M. Francais seems to have been aware of this difficulty, inasmuch as he speaks of the *sum* of directed lines only as a consequence of his first two theorems. Still, I do not object to admitting this condition as an essential characteristic of ϕ; but in that case the complete definition of a directed line will be a definition *nominis non rei*, or, in other words, *directed line* will be the name of a certain analytic function of the length and direction of a right line. From this it unfortunately follows that we are no longer constructing imaginaries, but simply reducing them to the same analytic form. However, let us see what this function is. It is, in the first place, clear that the expression $\phi(a,\alpha) = a.e^{\alpha\sqrt{-1}}$ satisfies the three foregoing conditions. In fact, we have 1° $\phi(a,o) = a.e^{o\sqrt{-1}} = a$, $\phi(a,\pi) = a.e^{\pi\sqrt{-1}} = a(\cos\pi + \sqrt{-1}\sin\pi) = -a$; 2° the equation $\phi(a,\alpha) = \phi(b,\beta)$ becomes $a.e^{\alpha\sqrt{-1}} = b.e^{\beta\sqrt{-1}}$, or, passing to logarithms, equating, and returning to numbers, $a = b$, $\alpha = \beta$; 3° the above proportion, by similar transformations becomes $\frac{a}{b} = \frac{c}{d}$ and $\alpha - \beta = \gamma - \delta$. But is this form $a.e^{\alpha\sqrt{-1}}$ the only one which satisfies these three conditions? I think not, and it seems to me evident that they will be equally true if we substitute an arbitrary coefficient for the imaginary $\sqrt{-1}$. So that the form $a.e^{\alpha\sqrt{-1}}$ will, in my opinion, only be a special case of the analytic expression for a directed line, in its conventional signification. Are there any other conditions which follow from this signification? To this question no answer is made, nor do I either see any.

Again, 4° the table of double argument which you (Gergonne) propose, as applied to a plane supposed to be so divided into points or *infinitesimal* squares that each square corresponds to a number which would be its *index*, would very properly indicate the length and position of the radii vectores which revolve about the point or central square corresponding to ± 0; and it is quite remarkable that if we designate the length of a radius vector by a, and the angle it makes with the real line $\ldots, -1, \pm 0, +1, \ldots$ by α, the rectangular co-ordinates of its *extremity remote from the origin* by x, y, the real line being the axis of x, the point would be determined by $x + y\sqrt{-1}$, and consequently, since $x = a\cos\alpha$, $y = a\sin\alpha$ by $a, a.e^{\alpha\sqrt{-1}}$. Thus we have a new *geometrical interpretation* of the function $a.e^{\alpha\sqrt{-1}}$ which, it seems to

me, is of more value than that of MM. Argand and Francais; but certainly we should not thereby conclude that this was a new method of constructing, *geometrically*, imaginary quantities, for the above indices presuppose them. However this may be, it is clear that your ingenious tabular arrangement of numerical magnitudes may be regarded as a central slice (*tranche centrale*) of a table of triple argument representing points and lines in tri-dimensional space. You would doubtless give to each term a tri-nomial form; but what would be the co-efficient of the third term? For my part I cannot tell. Analogy would seem to indicate that the tri-nominal should be of the form $p\cos\alpha+q\cos\beta+r\cos\gamma$, α, β and γ being the angles made by a right line with three rectangular axes, and that we should have $(p\cos\alpha+q\cos\beta+r\cos\gamma)(p'\cos\alpha+q'\cos\beta+r'\cos\gamma)=\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$. The values of p,q,r,p',q',r' satisfying this condition would be *absurd*; but would they be imaginaries, reducible to the general form $A+B\sqrt{-1}$?

On this letter Hamilton remarks in his *Lectures on Quaternions*, (Preface, p. 57), "The six *non-reals* which thus Servois with remarkable sagacity *foresaw*, without being able to *determine* them, may now be identified with the then unknown symbols $+i$, $+j$, $+k$, $-i$, $-j$, $-k$ of the quaternion theory;" and it may here be interesting to quote (*North British Review*, 1866), from a letter of Hamilton on the discovery of these symbols:

Oct. 15, '58.

"P. S.—To-morrow will be the fifteenth birthday of the Quaternions. They started into life, or light, full grown, on the 16th of Oct., 1843, as I was walking with Lady Hamilton to Dublin, and came up to Brougham Bridge, which my boys have since called the Quaternion Bridge. That is to say, I then and there felt the galvanic circuit of thought to *close;* and the sparks which fell from it were the *fundamental equations between i, j, k; exactly such* as I have used them ever since. I pulled out, on the spot, a pocketbook, which still exists, and made an entry, on which, *at the very moment*, I felt that it might be worth my while to expend the labor of at least ten (or it might be fifteen) years to come. But then, it is fair to say that this was because I felt a *problem* to have been at that moment *solved*—an intellectual want *relieved*—which had *haunted* me for at least *fifteen years before*. *Less than an hour* elapsed before I had asked and obtained leave of the Council of the Royal Irish Academy, of which society I was at that time president, to *read*, at the *next general meeting*, a *paper* on Quaternions, which I accordingly *did* on Nov. 13th, 1843."

It is also proper here to add a disclaimer from Gergonne as to any thought of the extension of his table to tri-dimensional space, until after the appearance of Argand's and Francais' papers; and that even then he saw no way by which to effect that result.

The above letter from M. Servois called forth a reply from Francais (Annales, Vol. IV., p. 364–367), and a third paper from Argand (Annales, Vol. V., p 197–209). In the former, Francais sustains Gergonne, who had already said that Servois asked too much of the new theory, demanding rigorous demonstrations of that which, as in the early history of negative quantities or the calculus, was perceived by a sort of instinct, the proofs of whose fundamental principles the earlier writers were not in a state to produce. He then adds a few examples of the facility with which one might pass from the proposed to the ordinary notation.

The equation of a triangle whose base coincides with the axis of reference is $a_\alpha+b_{-\beta}=c$, whence

$$a\cos\alpha+b\cos\beta=c, \text{ and } a\sin\alpha-b\sin\beta=o,$$

or, taking the sum and difference of the squares

$$a^2+b^2+2ab\cos(\alpha+\beta)=c^2,$$
$$a^2\cos 2\alpha+b^2\cos 2\beta+2ab\cos(\alpha-\beta)=c^2.$$

The equation of the circle referred to the center is $a_\phi=x+y\sqrt{-1}$, whence

$$a\cos\phi=x,\ a\sin\phi=y,\ x^2+y^2=a^2.$$

The equation of a circle referred to a diameter is $\rho_\phi+\sigma_{\frac{1}{2}\pi-\phi}=2a$, whence

$$\rho\cos\phi+\sigma\sin\phi=2a,\ \rho\sin\phi-\sigma\cos\phi=o,$$
$$\rho^2=2a\rho\cos\varphi,\ x^2+y^2=2ax.$$

The equation of an ellipse referred to the focus is $\rho_\phi+(2a-\rho)_\Psi=2e$, whence

$$\rho\cos\phi+(2a-\rho)\cos\Psi=2e,$$
$$\rho\sin\phi+(2a-\rho)\sin\Psi=o,\ \rho=\frac{a^2-e^2}{a-e\cos\phi}.$$

The reply of Argand is appended.

The new theory of imaginaries, already referred to several times in this publication, has two distinct and independent objects; it seeks, first, to render intelligible certain expressions whose presence in analysis has been inevitable, but which have not yet been referred to any known evaluable quantity; and, second, it presents a method, or a particular notation which employs geometric symbols concurrently with the ordinary algebraic signs. Hence,

from this double point of view, two questions arise: Has it been rigorously shown that $\sqrt{-1}$ represents a line perpendicular to those denoted by $+1$ and -1? Can the notation of directed lines furnish, in certain cases, demonstrations and solutions preferable either for their simplicity or brevity, etc., to those which they are intended to replace?

The first of these will, perhaps, always be open to discussion so long as we seek to establish the meaning of $\sqrt{-1}$ by analogy, from the commonly received ideas on positive and negative quantities and their ratios. Negative quantities have been and are still the subject of discussion; it will, therefore, be all the easier to raise objections to the new theory of imaginaries. But this difficulty will vanish if, with M. Francais, we *define* what is meant by a ratio of magnitude and position between two lines. Indeed, the relation between two such lines may be conceived of with all necessary precision. Whether this relation be called *ratio* or something else, it may always be made the subject of exact reasoning, and its consequences, in analysis and geometry, of which M. Francais and myself have given some examples, may be traced. The only remaining question, then, is whether it is proper to designate this relation as a *ratio* or *proportion*, words which already possess, in analysis, a determinate and fixed meaning. Now, this is permissible, because the new meaning is an *extension*, not a *contradiction*, of the old one. The latter is so generalized that the ordinary meaning becomes, so to speak, a particular case of the new one. There is then, no question here of *demonstration*.

Thus, for the analyst who first wrote $a^{-n}=\frac{1}{a^n}$, this equation was a definition of negative exponents, not a proposition proved or to be proved. All that it was incumbent upon him to show was that this definition was only a generalization of that of positive exponents, the only ones before known, and so for fractional, irrational and imaginary exponents. It has been said that Euler proved $(\sqrt{-1})^{\sqrt{-1}}=e^{-\frac{1}{2}\pi}$. The word *proved* may be exact if we mean that this equation is derived from $e^{x\sqrt{-1}}=\cos x+\sqrt{-1}\sin x$, which is readily shown to be the case; but it is not so as regards this latter; for to show that a certain expression has a definite value, implies the previous definition of the expression. But is there any definition of imaginary exponents antedating the so called demonstration of Euler? It seems not. When Euler sought to evaluate $e^{x\sqrt{-1}}$, he naturally resorted to the theorem $e^z=1+\frac{z}{1}+\frac{z^2}{1.2}+\ldots$ previously demonstrated for all *real* values of z. By making $z=x\sqrt{-1}$ he found

$$e^{x\sqrt{-1}}=1+\frac{x\sqrt{-1}}{1}-\frac{x^2}{1.2}-\ldots;$$

whence he concluded, not that $e^{x\sqrt{-1}}=\cos x+\sqrt{-1}\sin x$, but that, if the expression $e^{x\sqrt{-1}}$ was defined as representing a quantity equal to $\cos x+\sqrt{-1}\sin x$, we should thereby bring both real and imaginary exponents under the same law. Here again, then, we have the extension of a principle, not the demonstration of a theorem.

It is also by an extension of principles that I was led to regard $(\sqrt{-1})^{(\sqrt{-1})}$ as representing a perpendicular to the plane ± 1, $\pm\sqrt{-1}$. The two results conflict, and I certainly have not insisted upon my own; I only wish to observe that MM. Francais and Servois have attacked it from considerations which are after all of the very nature of those on which I relied to establish it.

But if the above perpendicular cannot be expressed by $(\sqrt{-1})^{\sqrt{-1}}$, how then shall it be represented? Or, rather, can any expression be found, whose adoption, as the representative of the perpendicular, shall bring all directed lines whatever under a common law, as is already the case for every line of the plane ± 1, $\pm\sqrt{-1}$? This is a question which must be of interest to geometers, at least to those who admit the new theory. To return to the original question, I observe that whether $\sqrt{-1}$ does or does not represent the perpendicular on ± 1 must depend upon the meaning of the word *ratio*; for it is agreed by all that

$$+1:\sqrt{-1}::\sqrt{-1}:-1 \text{ or that } \frac{\sqrt{-1}}{+1}=\frac{-1}{\sqrt{-1}}.$$

So that M. Servois' objection to Francais' proof of his first theorem, viz: "That it is not proved that $\pm a\sqrt{-1}$ is a mean, as to position, between $+a$ and $-a$" is equivalent to the assertion that the word *ratio* has no reference to position. In its usual acceptation, this is true; and on the other hand, it may be said that, in the conception of a ratio between quantities with different signs, the signs must be regarded. In the new meaning, direction *and* magnitude make up the idea of ratio. It is thus seen to be a question of words, decided by the exact definition given by Francais, which is an extension of the usual one.

The second point under discussion is more important. Doubtless no truth is reached by the notation of directed lines which cannot be attained by ordinary methods; but which method is the simplest? This question is, I think, worthy of examination. It is to the influence of methods and notations on the progress of the science, that modern mathematics owes its superiority. So that when anything new of this kind appears, we may at least examine it in this respect. Since the publication of the new theory, M. Servois alone has expressed an opinion on this point, and his opinion is not favorable to the new notation. Analytic formulæ seem to him more simple and expeditious. I would, however, claim for my method a more careful examination. I admit that it is novel, and that the mental operations it requires, although quite simple, demand some familiarity in order that they may be performed with the ease which follows practice in the ordinary operations of

Algebra. Some of the theorems I have proved seem to me easier than the corresponding purely analytic processes. This is, perhaps, an author's illusion, and I will not insist upon it; but I claim with more confidence the superiority of the method of directed lines for the demonstration of the Algebraic Theorem: "Every polynomial $x^n+ax^{n-1}+\ldots$ is decomposable into factors of the first or second degree." I feel it necessary to resume this demonstration, not only to reply to the objections of M. Servois, but also to show more fully how easily it is derived from the new principles. The importance and difficulty of this theorem, which has tasked the skill of the best geometers will, I think, excuse, in the eyes of the reader, some repetition. The demonstrations previously given may, I believe, be classified under two heads: Those of the first-class depend on certain metaphysical principles relating to the transformation of functions, which are doubtless true in themselves, but which, properly speaking, are not susceptible of rigorous proof. They are a sort of *axioms* whose truth cannot be appreciated unless we already grasp the *spirit* of Algebraic analysis; whereas to admit the truth of a *th orem*, it is sufficient to know the principles of this analysis; that is, to understand its definitions and language. Hence demonstrations of this kind have been frequently attacked. The *Recueil*, in which these remarks appear, offers several examples, and the appearance of such discussions is an indication of the fact that such reasoning is not above reproach.

In other cases the proposition to be established is approached directly, by showing that there is always at least one quantity of the form $a+b\sqrt{-1}$, which, when substituted for x, renders the polynomial zero; that is to say, that this polynomial may always be resolved into factors of the first or second degree. This is the method of Lagrange. This great geometer has shown that the previous methods of d'Alembert, Euler, Foncenex, etc., are inadequate (Résolution des equations numériques. Notes IX. and X). Some of them resorted to series, others to auxiliary equations; but they did not prove, as they should have done, that the co-efficients of these series and equations were always real. These geometers admit implicitly the principle that "if a problem involving an unknown quantity can be resolved in n ways, it must lead to an equation of the *nth* degree." Lagrange himself regards this legitimate, although he does not use it in the above-cited demonstrations. Now, may it not be said that this principle, probably true as it is, is not demonstrated, and belongs to that class of axioms above referred to? Especially would it seem as if this principle, which in theory is among the first to be demonstrated, was out of place, dependent as it is upon no little familiarity with the practice of the science. This remark is not a mere quibble, which, as regards conceptions deserving the respect of all geometers, would be as out of place as it is useless, but is made simply to show the difficulty in the way of a satistory treatment of this subject.

It would appear from the above that a demonstration at once simple, direct and rigorous is worthy the attention of geometers. I shall, therefore, resume that of my previous paper; but, to avoid all ambiguity, shall free it from any consideration of vanishing quantities. It will be convenient to restate briefly the first principles of the theory of directed lines.

Having taken $\overline{KA}$ as the direction of positive quantities, the opposite direction $\overline{AK}$, will be, as usual, that of negative quantities. Drawing the perpendicular $\overline{BKD}$ through K, one of the directions $\overline{KB}$, $\overline{KD}$, the former say, will correspond to imaginaries of the form $+a\sqrt{-1}$, the latter to those of the form $-a\sqrt{-1}$. The line drawn above the letters indicates that direction is considered, and, when we are only concerned with length, is suppressed. Assuming arbitrarily the points F, G, H, ... , P, Q, we have $\overline{FG}+\overline{GH}+\ldots+\overline{PQ}=\overline{FQ}$. This is the law of addition. If, between four lines, there exists the relation $\frac{AB}{CD}=\frac{EF}{GH}$, and, in addition, the angle between $\overline{AB}$, $\overline{CD}$ is equal to that between $\overline{EF}$, $\overline{GH}$, these lines are said to be *in proportion*. Hence the law of multiplication; for a product is merely a fourth term in a proportion whose first term is unity.

It is to be observed that these two rules are independent of any opinion one may have on the new theory. If it is desirable that $\sqrt{-1}$, a symbol to which the practice of Algebra continually gives rise, and which, sometimes called absurd, has yet never given absurd results, if it is desirable I repeat that this symbol should remain meaningless, while still not being zero, this will give rise to no difficulty. Directed lines will only be the *symbols* of numbers of the form $a+b\sqrt{-1}$. The above rules will be none the less true, but instead of deducing them *a priori* from purely metaphysical considerations, the first will depend on a simple construction. The second will be an immediate consequence of the formulæ $\sin(a+b)=\sin a\ \cos b+\ldots$; and therefore the use of these rules may give demonstrations entirely satisfactory.

Directed lines will then be symbols of the numbers $a+b\sqrt{-1}$. Like them they are susceptible of increase, decrease, multiplication, division, etc.; they will, as it were, correspond throughout, function for function, and, in a word, *represent* them completely. Hence, from this point of view, concrete quantities will represent abstract numbers; but conversely abstract numbers cannot represent concrete quantities. In what follows, the accents and subscripts are used to indicate the absolute magnitude of the quantities to which they are affixed; thus, if $a=m+n\sqrt{-1}$, m and n being real, it is understood that a_1 or $a'=\sqrt{m^2+n^2}$. Let then

$$y_x=x^n+ax^{n-1}+bx^{n-2}+\ldots+fx+g$$

be the proposed polynomial, n being a whole number; $a, b, \ldots f, g$ may be of the form $m+n\sqrt{-1}$. We are to prove that we

may always find a quantity such that, substituted for x, $y_x=o$. The polynomial may be constructed for any value of x by the preceding rules. Taking K as the initial point, P as the final one, $\overline{KP}$ will express this polynomial, and it is to be shown that x may be so determined as to cause P to coincide with K. Now, if among all the possible values of x, there is no one of which this is true, the line $\overline{KP}$ cannot become null; and of all the values of $\overline{KP}$ there will necessarily be one smaller than the rest. Designate this minimum value of x by z; then $y'_{(z+i)}<y'_z$ cannot be true, whatever the value of i. Now, developing, we have

$$(A)\quad \begin{cases} y_{(z+i)}=y_z+ \\ \qquad [nz^{n-1}+(n-1)\alpha z^{n-2}+\ldots.+f]i+ \\ \qquad \left\{\frac{n}{1}\cdot\frac{n-1}{2}z^{n-2}+\ldots.\right\}i^2+\ldots. \\ \qquad\qquad +(nz+a)i^{n-1}+i^n. \end{cases}$$

As the co-efficients of the several powers of i may become zero, and this is a special case, it is better to replace the above equation by

$$(B)\quad y_{(z+i)}=y_z+Ri^r+Si^s+\ldots.+Vi^v+i^n;$$

and so make the solution general; R, S and V not being zero and the exponents $r, s, \ldots, v, n$ being increasing. Observe that if all the coefficients of (A) were zero, the equation would reduce to $y_{(z+i)}=y_z+i^n$. Making then $i=\sqrt[n]{-y_z}$, we shall have $y_{(z+i)}=o$, and the theorem would be established for this case, which in what follows may therefore be set aside. We shall then suppose that the second member of (B) has at least three terms. With this premise, construct $y_{(z+i)}$, taking $\overline{KP}=y_z$, $\overline{PA}=Ri^r$, $\overline{AB}=Si^s$, $\ldots.$, $\overline{FG}=Vi^v$, $\overline{GH}=i^n$; we shall have $y'_z=\overline{KP}$, $R'i_1^r=PA$, $Si_1^s=AB$, $\ldots.$, $V'i_1^v=FG$, $i_1^n=GH$; for evidently, in general, $p'q'=(pq)'$.

$y_{(z+i)}$ will be represented by the broken or straight line

$$KPAB\ldots FGH, \text{ or by } \overline{KH};$$

and it is to be proved that we have $KH<KP$.

Now the quantity i may vary in two ways:

1°. In *direction*; and it is clear that if it varies by an angle α, its power i^r will vary by an angle $r\alpha$. Let then α be the angle by which $\overline{PA}=Ri^r$ is greater than $KP=y_z$. If i is made to vary by the angle $\frac{\pi-\alpha}{r}$, $\overline{PA}$ will vary by the angle $\pi-\alpha$; that is, the direction of $\overline{PA}$ will become opposite to that of $\overline{KP}$; so that the point A will be found on the line PK, prolonged, if necessary, through K.

2°. The direction of i being supposed fixed, we may, in the second place, cause it to vary in *magnitude*; and first, if $PA>KP$, we may diminish i till $PA<KP$, so that A will fall between K and P. Then, if the magnitude of i, so diminished, is not such that $R'i_1^r>S'i_1^s+\ldots.+V'i_1^v+i_1^n$, we may, by diminishing it still further, make this inequality true, for the exponents $s, \ldots., v, n$ are all greater than r. Now this inequality is equivalent to $PA>AB+\ldots.+FG+GH$; therefore the distance AH will be less than PA, and, consequently, if we describe a circle with A as a center and radius AP, the point H will lie within this circle, and it follows from elementary geometry, that K being on the prolongation of the radius PA, in the direction of the center A, we shall have $KH<KP$.

To follow this demonstration, I would ask the reader to make the diagram. By the application thereto of the above cited simple fundamental principles, it will be seen that, with the exception of the development (A) which is algebraic, the remainder of the demonstration is made, as it were, at sight, without any mental effort.

It is almost superfluous to dwell upon an objection which might be made to what precedes, namely, that if one undertook to diminish the value of x by the method prescribed for diminishing y'_x, one might never succeed, because the value of i, in the successive substitutions, might diminish by constantly decreasing quantities. Indeed, the contrary is not proved; but from this it only follows that the above considerations cannot furnish, at least without new developments, an approximative method, and this does not in the least invalidate the demonstration of the theorem. M. Servois' objection is easily answered. "It seems to me," he says, "that it is not enough to find values of x which render the polynomial constantly less; it is necessary, in addition, that the law of diminution should necessarily reduce the polynomial to zero, or, if I may use the expression, such that zero is not the *asymptote* of the polynomial." It has been proved that we may not only find for y'_x constantly diminishing values, but a value less than any assignable one. If the polynomial cannot be reduced to zero, its least value will then be other than zero, and in this case also the demonstration holds good. The close of M. Servois' sentence would seem to indicate that he makes a distinction between an infinitely small limit and one which is absolutely zero. If such was his meaning he might be answered in the words of M. Gergonne . . . . Doubtless M. Servois' difficulty arises from the equation of the hyperbola $y=\frac{1}{x}$. It is unquestionably true that in this equation, although we may assign to y a value less than any assignable one, y cannot become zero unless x is supposed infinite. But this is not the case in our demonstration; for certainly it is not an infinite value for x which will render the polynomial y'_x zero.

Let us now resume the question which has given rise to the above explanations. It may be asked if it is possible to translate what precedes into the ordinary language of analysis. It seems to me quite probable, although it may be difficult in this way to obtain so simple a result. To effect this it would seem necessary to assimilate the notation of imaginaries to that of directed lines, writing, for instance:

$$\sqrt{a^2+b^2}\left\{\frac{a}{\sqrt{a^2+b^2}}+\frac{b}{\sqrt{a^2+b^2}}\sqrt{-1}\right\}$$

for $a+b\sqrt{-1}$;

$\sqrt{a^2+b^2}$ might be called the *modulus* of $a+b\sqrt{-1}$, and would represent the absolute length of the line $a+b\sqrt{-1}$, while the other factor, whose modulus is unity would denote its direction. We should only prove 1° that *the modulus of the sum of several quantities is not greater than the sum of their moduli*, which is equivalent to saying that the line AF is not greater than the sum of the lines AB, BC, ... EF; 2° *that the modulus of the product of several quantities is equal to the product of their moduli.*

The further investigation of the relations between the notations I must leave to those more skillful than myself. If this attempt to obtain a purely analytic demonstration as simple as that derived from the new theory is successful, analysis will be the gainer in thus reaching, by an easy method, a result whose difficulties were not unworthy the notice of Lagrange himself. If, on the contrary, this attempt should prove unsuccessful, the notation of directed lines will retain an evident advantage over the ordinary one; and in either case the new theory will have rendered some small service to science.

In closing, I may be permitted to make a remark on a note from M. Lacroix (Annales, Vol. IV, p. 367). This learned professor says that the Philosophical Transactions of 1806 contain a memoir from M. Buée on the very subject of which M. Francais and myself have written. Now, it was in this same year that my essay appeared, a pamphlet in which I explained the principles of the new theory, and of which the paper inserted in Vol. IV of the *Annales* (p. 133) is but an extract; it is well known, too, that the publications of academies can appear only after the date which they bear. This is sufficient to prove that if the contribution of M. Buée was wholly his own, as is quite possible, it is also quite certain that I could have had no knowledge of his paper when my treatise appeared.

In the foot notes to the Preface of Hamilton's Lectures on Quaternions, the reader will find full references to the labors of other writers on this subject, including Warren (1828), Peacock, Ohm, Mourey (1828), Gauss, Buée (1806), Gompertz (1818), Carnot, Wallis (1685), MacCullagh, Argand, Francais, Servois, Grassmann, DeMorgan, Graves, De-Foncenex, Euler, etc. While giving full credit to the results of his predecessors and co-workers, Hamilton justly claims to be alone the founder of a *system*. Moreover, the fundamental conception of this system was radically different from those entertained by previous writers. In the latter inclined or perpendicular lines to the plus and minus axis were represented by imaginaries, whereas *all* unit lines in space are represented by Hamilton by distinct square roots of negative unity, they being all real. No *one* direction is assumed positive, nor is any system of reference chosen independent of the lines of the construction involved in any special problem.

In addition to the works of Hamilton, Tait and Kelland, may be especially mentioned the *Calcolo dei Quaternioni, Bellavitis, Modena*, 1858, and the original paper (Memoirs of the Italian Society, 1854), by the same author, which has also been translated from the Italian into French by Laisant (*Exposition de lá Méthode des Equipollences. Paris*, 1874); the *Théorie Elementaire des Quantités Complexes*, by Hoüel, Paris, 1874; the *Fonctions doublement périodiques*, of MM. Briot and Bouquet, and a treatise by Allégret, *Sur le Calcul des Quaternions de M. Hamilton*. Paris, 1862.

As possessing some historic interest may be added, in addition to the works cited in the above-mentioned Preface, Truel, 1786, referred to by Cauchy, Woodhouse (Phil. Trans. 1801), Khun, (Nouveaux Mémoires de Petersburg, Vol. 3), and *Le Calcul Directif*, a series of articles by Transom, in the *Nouvelles Annales de Mathématiques*, 1868.

A. S. HARDY.

ROUSSET'S DEEP SEA SOUNDER.—This device is chiefly novel in dispensing with the usual sounding line altogether, and therefore with the trouble of paying out and hauling in. It is the invention of a Russian engineer, M. Paul C. Rousset, of St. Petersburg, and consists of an ordinary registering log having an inflated balloon or float attached to its upper end, and a lead or sinker hung by a trigger from its bottom. The log is lowered into the sea and drops downward through the water in a vertical position, the sinker being underneath, and the balloon uppermost. In its descent the vanes revolve, and the indicator registers the depth. When it reaches the bottom the sinker is detached by the loose trigger, and the buoyancy of the balloon is such that it raises the log again to the surface, where it floats until it is picked up by the ship.

ON THE CALCULATION OF DIMENSIONS AS DEPENDING ON THE ULTIMATE WORKING STRENGTH OF MATERIALS.

By Dr. J. WEYRAUGH, Professor at the Polytechnicum, Stuttgart. Translated by GEORGE RUDOLPH BODNER, Assoc. M. Inst. Civil Engineers.

From Selected Papers of the Institution of Civil Engineers.

I. LEADING POINTS OF VIEW.

In the choice of the admissible stresses for iron and steel either the ultimate strength or the limit of elasticity has been the starting point. The new method aimed at by German engineers depends on the ultimate strength only. There are no longer any representatives of the other school among German writers, although the stress produced by the working load is always kept below the ordinary limit of elasticity. Formerly the ultimate working strength* a (*Arbeitsfestigkeit*) was assumed as constant, so that a bar which had once sustained a certain static load, could also support it under a quicker application and with an indefinite number of repetitions. The question is, whether in the present state of knowledge a may not be expressed with smaller errors by a formula. If a formula is decided upon, the general algebraical expression for it,

$$a=f(xy\ldots), \quad . \quad . \quad . \quad . \quad (1)$$

denoting that a is some function of x, y, . . . takes the place of the constant working strength

$$a=C,$$

where C is some constant used in the earlier method of determining dimensions. Various general formula for a have been proposed in Germany. Whichever of them be adopted the choice of a numerical expression may turn out to be different according to the material and experience of the individual, just as until now different values of $a=C$ have been in use. As, however, under the most favorable circumstances, no values universally correct can result from a formula, having regard to all unforeseen and uncontrollable influences (faulty material, unintentional distribution of stress, rust, shocks, &c.), so now, as formerly, factors of safety must be used, the choice of which depends upon the object of the structure and the judgment of the designer. In this way, even with the same numerical value of the working strength a different values are obtained for the admissible stress, just as when for the same assumed value of a, say 3,500 kilogrammes per square centimeter, different engineers chose 600, 700, and 800 as admissible stress. When the admissible stress per unit of area has been fixed, the necessary effective area of section is made, as hitherto,

$$F=\frac{\max B}{b}, \quad . \quad . \quad . \quad . \quad . \quad (2)$$

max B being understood to mean the numerical value of the absolute maximum load.

The desired improvement (*Neuerung*) requires, as compared with the old method, only an altered choice of b.

II. WÖHLER'S LAW.

That under a frequent alternation of tension and compression a bar does not sustain the same stress as under a static load has probably long been everywhere appreciated. The same remark applies to variable stress, whether tensile alone or only compressive. Before Wöhler's time account had been already frequently taken of this in calculations.*

The formula given by German engineers aim mainly at substituting, instead of a mere guess (*Schatzung*) at the most suitable stress on every occasion, a certain consistency or regularity.

* Or resistance to fracture under the straining action produced by the working conditions of the material. See definition in IV.

* *Vide* under X., and Weyrauch, "Strength and Determination of the Dimensions of Structures of Iron and Steel." Translated by Prof. Jay du Bois, New York, 1877, Wiley, pp. 2 and 159.

From the nature of the case no far-reaching accuracy can be attempted, but only a limitation of previous errors. Former rules depended more upon "feeling" than on definite experimental results. Still it was considered that they were to be preferred to the assumption of a constant working strength. Thus arose the recognition accorded to the experiments undertaken by Wöhler between the years 1859–70 on behalf of the Prussian Ministry of Commerce, in order to determine the capability of materials for resistance to repeated and alternating straining actions. A complete account of these experiments, is found is Wöhler's publications,* and Spangenberg, on continuing the experiments, confirmed his results.† In earlier experiments on the strength of materials, it was ascertained by a gradually increasing load what tensile, compressive or shearing stress, once for all, could break a bar whose sectional area was unity, and this quantity t was called the tensile, compressive, shearing or breaking strength of the material. By Wöhler's experiments it was shown, that, though a given stress t can break the piece by a single application, smaller stresses can produce fracture if they are often enough repeated.

Wöhler concluded from this that the change in the grouping of the molecules, which is produced by the varying stress, acts unfavorably on the resistance of the material. In that case fracture must take place more readily the greater the differences in the alternate stresses, because with these the change of position of the molecules also increases. With reference to these considerations, and on the bases of his experimental results, Wöhler enunciated the following law:

The fracture of a material may be effected not only by a static load producing stress equal to t, but also by often repeated vibrations producing stresses of which no one attains t. The differences of stress in so far affect the cohesion, that, as they increase, the minimum stress capable of causing fracture decreases. With the stress t the material is broken by a single application. Smaller stresses than t can produce fracture by many repetitions. The smaller the stress, the more numerous the repetitions necessary for fracture. Conversely, the stress may be greater, the fewer repetitions it is intended to make. According to these principles, in judging of the degree of safety of a structure, much depends on whether it is only expected to remain in use for a certain time, or whether it is intended for permanent durability.

III. REMARKS ON WÖHLER'S LAW.

That, since the experiments of Fairbairn,* Wöhler, and Spangenberg, apart from earlier experience, the ultimate working strength a can longer be considered constant, will scarcely be disputed. It is another question whether Wöhler's law forms a reliable basis for judgment. In Germany this has been pretty generally assumed, as the numerous methods of determining dimensions based upon it testify, of which Gerber's† has been adopted by the Bavarian Government.

The Launhardt-Weyrauch method, which has probably found the most general acceptance, also starts from Wöhler's law. If this law is recognized, it must still be admitted that further definition of it is desirable.

In Wöhler's experiments the load was applied at very brief intervals; nevertheless, the stresses required a certain time to attain their full magnitude; but there is no question of actual blows (*Stosse*). Now, what influences are respectively and separately due to the frequency of the application, the rate of increase, and the duration of the individual straining actions? The two latter influences have been neglected under the assumption of a constant ultimate working strength; in bridge design the first is ordinarily viewed too unfavorably, if considered in the light of Wöhler's results.

That the law in question is not absolutely essential to the explanation of a variable strength has been lately shown

* Wöhler in Zeitschrift für Bauwesen, 1860, 1863, 1866, 1870, *passim*.
† Spangenberg in Zeitschrift für Bauwesen, 1874, 1875, and Minutes of Proceedings Inst. C. E., vol. lx., p. 415.

* *Vide* Phil. Trans. 1864, p. 311, "Useful Information for Engineers," 3rd Series, 1866.
† Gerber in Zeitschrift des Bayerischen Arch. und Ing-Vereins, 1874.

by Lippold.* He starts from the following law: "To break a bar a certain amount of mechanical work is necessary, and this can be accumulated in the material at once, just as well as by repeated loads. These loads must, however, be applied instantaneously, or so rapidly that vibrations arise." It is essential to this view, that only by means of vibrations can equal strain (*i. e.*, proportionate extension, or proportionate shortening, as the case may be), and with it equal stress and equal work, be attained by a load below that which produces t as by t itself, while less weight is attached to the repetitions of the straining action. Lippold arrives at formula for a, according to which the strength varies in the same manner as when based on Wöhler's law.

It is doubtless possible that in time, through speculation and experiment, safe foundations for a correct theory of properties relating to strength will be reached. The author, however, is not satisfied to rest contented till then with the certainly false assumption of constant strength. Smaller errors and greater safety for structures can already be attained through the results of Wöhler's experiments.

IV. ON THE STRENGTH OF MATERIALS.

To start from Wöhler's law, regarding it as purely empirical:

The words "often repeated" (II.) may be suppressed, and it may be left an open question what influences are concerned in the fracture of the piece. If, for the method hitherto used in determining dimensions, a new method is to be substituted, the latter must be sufficiently well founded, simple in its application, and in conformity with experience. At the same time it may be remembered that it is not a question of ingenious laws of unattainable accuracy, but of a useful guide for practical purposes. In the first place it has to be determined by a formula how the ultimate working strength a varies with the difference between successive stresses.

In developing or judging of this formula the results of experiment should find application, but if possible, only such as have been obtained with the same class of material and under exactly the same circumstances; for only in this way can the effects of varied loading be ascertained independently of foreign influences.

The variable resistance to fracture, a, is in Germany called, according to Launhardt, "ultimate working strength" (*Arbeitsfestigkeit*). The terms, too, of tensile strength, compressive strength, shearing strength, &c., are to be taken in a wider sense than hitherto, to mean the ultimate working strength a for tension, compression, shearing, &c. Three special values of the ultimate working strength are of particular interest.

I. The amount per unit of section of the resistance to fracture by a single application of a statical, or at any rate very gradually applied load, which hitherto has been alone considered, may be called "statical breaking strength," t, (*Tragfestigkeit*).

II. If, after every application of the load, the bar reverts to its original unloaded condition, and if all stresses are in the same sense,* *e. g.*, tension, compression, or shearing in one sense only, the greatest stress which can be sustained under the specified number of repetitions is called, according to Launhardt, the "primitive strength," u, (*Ursprungsfestigkeit*).

III. Finally, for strength as regards alternating stresses of equal intensity in opposite senses, *i. e.*, for tension and compression, or for shearing in opposite senses, these being due to vibrations taking place about the original position of unstrained equilibrium, the author has introduced the term "vibration-strength," s, (*Schwingungsfestigkeit*).

Of course the statical breaking strength t and the primitive strength u apply to either tension, compression, or shearing. If it be assumed, with Wöhler, that repetitions of the straining action are essentially dangerous to the material, u and s must be less the more

* *Vide* Organ für die Fortschritte des Eisenbahnwesens, 1879.

* "Direction" of a force, or a stress, is any line parallel to its line of action, and may be regarded indifferently as from left to right or right to left. "Sense" distinguishes between these two alternatives. Thus, if a force is exerted from right to left it is exerted in one sense; if from left to right, in the opposite sense. The two opposite senses may, if required, be distinguished as positive sense and negative sense. This use of "sense" is universal among French writers on mechanics, and helps precision of statement.

repetitions are intended, and hence minima for a bridge which is to last for ever. In giving numerical values in the sequel the latter case is contemplated.

V. LAUNHARDT'S FORMULA.

Suppose a bar whose sectional area is unity to experience stresses in the same sense which vary between the maximum a and the minimum a'. The difference of stress is here

$$d=a-a',$$

and

$$a=d+a' \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

According to Wöhler's law a diminishes as d increases. From (3), and the definitions of the statical breaking strength t and primitive strength u, the two limits of value for a are

for $a'=0$, $a=d=u$,
" $d=0$, $a=a'=t$.

As according to Wöhler a should be a function of d,

$$a=fd \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where f is at present an unknown multiplier. It is known, however, that

for $d=0$ since $a=t$, $f=\infty$,
" $d=u$ " $a=d$, $f=1$.

For intermediate values of a the variation of f must be determined by considering experimental results obtained under circumstances, as nearly as possible, identical.

The coefficient chosen by Launhardt

$$f=\frac{t-u}{t-a} \quad . \quad . \quad . \quad (5)$$

corresponds to the best of these results, and fulfills the imposed conditions. Hence by (4)

$$a=\frac{t-u}{t-a}d=\frac{t-u}{t-a}(a-a');$$

and therefore, reducing,

$$a=u\left(1+\frac{t-u}{u}\cdot\frac{a'}{a}\right) \quad . \quad . \quad . \quad (6)$$

If B denotes the total load on a bar of any section, alternating between maximum and minimum values, the ratio of the less to the greater limit of stress is

$$\Phi=\frac{\min \mathrm{B}}{\max \mathrm{R}}=\frac{a'}{a}; \quad . \quad . \quad . \quad (7)$$

so that

$$a=u\left(1+\frac{t-u}{u}\cdot\frac{\min \mathrm{B}}{\max \mathrm{B}}\right) \quad . \quad . \quad (8)$$

The is Launhardt's formula.* It is applicable when a member of a structure is always subjected solely to tension, solely to compression, or solely to shearing stress in one sense, the values of t and u only, the statical breaking strength and the primitive strength for tension, compression, or shearing, as the case may be, having to be substituted.

VI. REMARKS ON LAUNHARDT'S FORMULA.

In order to ascertain whether Launhardt's choice of the coefficient f is applicable also to intermediate cases, equation (6) is to be solved for a, giving

$$a=\frac{u}{2}+\sqrt{\frac{u^2}{4}+(t-u)a'}, \quad . \quad . \quad . \quad (9)$$

where the + sign must be chosen before the radical, because a must be positive and not less than u; since, as defined in V., its least value is u, and greatest t.

The most complete results are those which Wöhler obtained† in making bending experiments with unhardened Krupp spring steel. This material showed for a statical breaking strength of about 1,100 centners per square inch (Rhenish), a primitive strength of $u=500$ centners per square inch; whence from equation (9) the ultimate working strength in this case must be

$$a=250+\sqrt{62{,}500+600\,a'}.$$

There follows for.....$a'=$	0	250	400	600	1,100
By Launhardt's equation.....$a=$	500	711	800	900	1,100
By Wöhler's experiments.....$a=$	500	700	800	900	1,100

For unhardened spring steel from Mayr of Werben the experiments also give $u=500$, $t=1{,}100$; while for hardened Krupp spring steel, the lowest values are $u=600$, and $t=1{,}200$.

* *Vide* Zeitsch. des Arch. und Ing-Vereins zu Hannover, 1873.

† Wöhler "Uber die Festigkeitsversuche mit Eisen und Stahl;" Berlin Zeitschrift für Bauwesen, 1870, pp. 7 and 14 of the reprint. As it is merely a question of comparison, the author retains Wöhler's original values and denominations.

Wöhler obtained from bending experiments with iron for axles made by the Phœnix Company $t=550$, $u=300$; whence follows

$$a=150+\sqrt{12{,}500+300\,a'}.$$

For example, if $a'=240$, equation (9) gives for the ultimate working strength $a=440$; which agrees exactly with the value given by his tension experiments.

Unfortunately only a few among Wöhler's experiments give t, u, s, and intermediate values of a. Considering, however, that absolutely exact laws for constructive materials will certainly never result from experiments, that even in brands of iron acknowledged to be good, differences in the statical breaking strength t of as much as 40 per cent. occur, and that it is merely a question of finding a substitute for the still more rough and incorrect assumption of a constant a, even the preceding might suffice for practical purposes until more facts are accumulated.

Launhardt's formula (4) is the expression of Wöhler's law. By the latter the limits of value for f are also determined. In the first instance, the choice of the interpolation formula for f is arbitrary. Nevertheless, this choice appears to be well justified by the only experiments of Wöhler's suited for comparison. Neither results from other sources, nor experience, nor practical feeling are against it. Even for more exact determinations than those under consideration such an approximation ought to be considered satisfactory. To the author's mind, it would appear sufficient if the deviations of the real values of a from those given by the formula did not exceed the deviations from one another of real values of a in good and commonly used materials.

VII. WEYRAUCH'S FORMULA.

Suppose a bar of unit sectional area subjected to alternating straining actions in opposite senses. If then a be the numerical value of the greater, a' that of the lesser limit of stress, the difference of stress is $d=a+a'$, and therefore

$$a=d-a' \quad . \quad . \quad . \quad . \quad (10)$$

According to Wöhler, a diminishes as d increases, and is a function of d, so that as in V.,

$$a=fd \quad . \quad . \quad . \quad . \quad . \quad (10\text{A})$$

Since, by the definitions of the primitive strength u, and the vibration strength s in IV.,

for $a'=0$, $a=u=d$,
" $a'=s$, $a=s=\frac{1}{2}d$,

therefore the following conditions must be fulfilled:

for $a=u$, $f=1$,
" $a=s$, $f=\frac{1}{2}$.

Intermediate cases should depend on the results of experiment which, however, at present do not exist. In the meantime the two conditions just mentioned are fulfilled by the coefficient

$$f=\frac{u-s}{2u-s-a}; \quad . \quad . \quad . \quad (11)$$

hence, from (10A),

$$a=\frac{u-s}{2u-s-a}\,d=\frac{u-s}{2u-s-a}(a+a'),$$

and therefore

$$a=u\left(1-\frac{u-s}{u}\cdot\frac{a'}{a}\right) \quad . \quad . \quad . \quad (12)$$

If, now, for a bar of any section, max B is the numerical value of the absolutely greatest load, max B′ the numerical value of the greatest load in the opposite sense, distinguished from the other as negative, then the ratio of the less to the greater limit of stress is

$$\Phi=-\frac{\max B'}{\max B}=-\frac{a'}{a}; \quad . \quad . \quad . \quad (13)$$

whence from (12)

$$a=u\left(1-\frac{u-s}{u}\cdot\frac{\max B'}{\max B}\right) \quad . \quad . \quad . \quad (14)$$

This is Weyrauch's formula. It is always to be applied in those cases where a member of a structure is subject to stresses alternating between tension and compression, or between shearing actions in opposite senses. By u and a are to be understood, respectively, the primitive strength and ultimate working strength for the numerically greater of the two alternate stresses if they differ in amount.

VIII. REMARKS ON WEYRAUCH'S FORMULA.

If equation (12) be solved for a,

$$a=\frac{u}{2}+\sqrt{\frac{u^2}{4}-(u-s)a'} \quad . \quad . \quad (15)$$

In this formulua, from (12),

$$(u-s)a'=au-a^2.$$

The maximum of this expression is found by differentiation with respect to a, thus:

$$\frac{d[(u-s)a']}{da}=u-2a=0,$$

to occur when $a=\frac{1}{2}u$, and the maximum value of

$$(u-s)a'=au-a^2=\frac{u^2}{4}.$$

Hence the expression under the radical in equation (15) can never become imaginary, and this refutes an objection in "Engineering" (vol. xxix., pp. 304, 341).

Wöhler found for alternations of tension and compression for "Phœnix" iron

$$u=2{,}190,\ s=1{,}170;$$

for Krupp cast steel for axles

$$u=3{,}510,\ s=2{,}050;$$

for shearing stress in opposite senses with the same material

$$u=2{,}780,\ s=1{,}610$$

centners per square inch (Rhenish). It is a decided defect that the choice of the coefficient f for the intermediate stages cannot for the present be checked by experiment. This is of course true of all empirical formulæ, which may be constructed on Wöhler's data for stresses in opposite senses. When, however, it is admitted that s and u are not equal, or in other words, that the same ultimate working stress cannot apply in cases of alternation between tension and compression that applies to intermittent tension alone, it becomes evident that an interpolation formula must be adopted. The author considered that it ought to be built up by reasoning similar to Launhardt's, and hence in Germany the formulæ,

$$a=u\left(1+\frac{t-u}{u}\Phi\right)\text{ for positive }\Phi\text{ .}\quad(16)$$

$$a=u\left(1+\frac{u-s}{u}\Phi\right)\text{ for negative }\Phi\text{ .}\quad(17)$$

are always applied side by side. In both (16) and (17) t, u, and s are numerical values without sign [+or−], while the ratio Φ of the least to the greatest limit of stress is positive or negative according as the extreme alternate stresses are in the same or different senses. The results which are obtained by formula (17) do not at any rate contradict practical feeling or assumptions hitherto customary (*vide* X).

IX. ADMISSIBLE STRESS FOR IRON.

In determining the numerical values of the admissible stress per unit of area, Wöhler's experimental results must be applied. At the same time every illusion as to the universal applicability of the latter is to be put aside; just as formerly a new series of experiments on the statical breaking strength t was accepted without their results being henceforth exclusively used. If, for instance, in such cases a statical breaking strength of $t=3{,}870$ kilogrammes per square centimeter resulted, 3,500 perhaps, was taken as a low value; so, if not still more cautiously, similar judgment must be exercised in the choice of a. Beyond this, the help of factors of safety was always resorted to, and this must still be so in the future. In the following determinations particular reference is made to iron bridges. For simple compression, in accordance with the usual practice, the same limiting stresses as for tension are adopted.

Alternating stresses, either exclusively tensile, or exclusively compressive.—In what follows the numerical intensities of stress are expressed in kilogrammes per square centimeter. With iron for axles of the "Phoenix" Company, bending experiments of Wöhler's gave $t=4{,}020$, $u=2{,}190$; whence $\frac{t-u}{u}=\frac{5}{6}$; and by (16)

$$a=u\left(1+\frac{t-u}{u}\Phi\right)=2{,}190\left(1+\frac{5}{6}\Phi\right).$$

If the formula is to afford the desired security for any tensile or compressive stress, a start must always be made from the most unfavorable case of this stress. For the same iron, with the direct application of tensile stress, $u=2{,}190$ also; but $t=3{,}290$ only. Therefore take

$$\frac{t-u}{u}=\frac{3{,}290-2{,}190}{2{,}190}=\frac{1}{2},$$

very approximately; and suppose the ultimate working strength be further reduced by rounding off the value of u on the side of safety, to

$$a=2{,}100\left(1+\frac{1}{2}\Phi\right).$$

If $\frac{1}{3}$ is chosen as a suitable factor of safety, the admissible stress per square centimeter then becomes

$$b=700\left(1+\frac{\Phi}{2}\right) \quad . \quad . \quad . \quad . \quad (18)$$

Alternation between tensile and compressive stresses.—For the Phoenix iron above-mentioned Wöhler found $u=2{,}190$, $s=1{,}170$; hence in this case $\frac{u-s}{u}=\frac{7}{15}$, and by (17),

$$a=u\left(1+\frac{u-s}{u}\Phi\right)=2{,}190\left(1+\frac{7}{15}\Phi\right).$$

Rounding off these values on the safe side by diminishing 2,190 to 2,100 and increasing $\frac{7}{15}$ to $\frac{1}{2}$, since Φ is negative, there results

$$a=2{,}100\ (1+\tfrac{1}{2}\Phi),$$

whence, with the factor of safety $\frac{1}{3}$ the admissible stress per square centimeter becomes

$$b=700\left(1+\frac{\Phi}{2}\right) \quad . \quad . \quad . \quad (19)$$

Shearing Stresses.—In most practical cases a and b for shearing stresses may be made $\frac{4}{5}$ of what they would be with the same ratio of stress Φ for tension and compression.* Therefore, generally,

$$b=560\left(1+\frac{\Phi}{2}\right) \quad . \quad . \quad . \quad (20)$$

where Φ is positive or negative, according as the alternating limits of stress are in the same or in opposite senses.

X. REMARKS ON THE CHOICE OF b.

It has been shown in the above that the stress per square centimeter may be generally expressed,

for positive Φ by $b=v\,(1+m\Phi)$. (21)
" negative Φ " $b=v\,(1+n\,\Phi)$. (22)

where v, m, n, are constants depending on the nature of the material and sense of stress, Φ the ratio of the less to the greater limiting stress. When the admissible stress per unit of area has been fixed, the necessary net sectional area is found by

$$F=\frac{\max B}{b}.$$

For iron, in particular, as explained in IX., there may be assumed for tension and compression $v=700$, $m=n=\frac{1}{2}$, so that very simply,

$$\left.\begin{aligned} b&=700\left(1+\frac{\Phi}{2}\right),\\ F&=\frac{\max B}{700\left(1+\frac{\Phi}{2}\right)}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (23)$$

By this threefold safety is attained if

$$t=3{,}150,\ u=2{,}100,\ s=1{,}050.$$

Smaller values of strength for bridges than these figures indicate are certainly not necessary, and have not been hitherto customary. Notwithstanding this, as has already been remarked in I., these values can have no universal application, and should they be thought too high for the material and the purpose, the following may, perhaps, be suitable: $v=640$, $m=n=\frac{1}{2}$; whence

$$\left.\begin{aligned} b&=640\left(1+\frac{\Phi}{2}\right),\\ F&=\frac{\max B}{640\left(1+\frac{\Phi}{2}\right)}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (24)$$

Different values also may be assigned to m and n; for steel, for instance, different values cannot be avoided.*

Exception† has been taken to the fraction $\frac{1}{2}$. With equal justice the choice of $b=700$ might have been considered dubious, and 687 preferred. It may suffice to answer that, in these objections, general and special facts have been confused.

When for booms of girders p is the dead load, q the total load per lineal meter, and $\Phi=\frac{\min B}{\max B}=\frac{p}{q}$ then

$$b=v\left(1+m\frac{p}{q}\right)=700\left(1+\frac{p}{2q}\right); \quad . \quad . \quad (25)$$

these latter numerical values for wrought iron being according to IX.

When the numerical values assumed for b, the admissible stresses for a static load, for alternations between tension and an unloaded condition, and for alternations between tension and compression of equal amount, would stand in

* *Vide* "Strength and Determination of the dimensions of Structures of Iron and Steel," by Dr. P. J. J. Weyrauch, pp. 81-88 *ante*.

* *Vide* Weyrauch "Strength and Determination," &c., pp. 60-68.
† *Vide* "Engineering," vol. xxix., p. 263.

In this formulua, from (12),

$$(u-s)a'=au-a^2.$$

The maximum of this expression is found by differentiation with respect to a, thus:

$$\frac{d[(u-s)a']}{da}=u-2a=0,$$

to occur when $a=\frac{1}{2}u$, and the maximum value of

$$(u-s)a'=au-a^2=\frac{u^2}{4}.$$

Hence the expression under the radical in equation (15) can never become imaginary, and this refutes an objection in "Engineering" (vol. xxix., pp. 304, 341).

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$$a=u\left(1+\frac{u-s}{u}\Phi\right)\text{ for negative }\Phi\text{ . (17)}$$

are always applied side by side. In both (16) and (17) t, u, and s are numerical values without sign [+or−], while the ratio Φ of the least to the greatest limit of stress is positive or negative according as the extreme alternate stresses are in the same or different senses. The results which are obtained by formula (17) do not at any rate contradict practical feeling or assumptions hitherto customary (*vide* X).

IX. ADMISSIBLE STRESS FOR IRON.

In determining the numerical values of the admissible stress per unit of area, Wöhler's experimental results must be applied. At the same time every illusion as to the universal applicability of the latter is to be put aside; just as formerly a new series of experiments on the statical breaking strength t was accepted without their results being henceforth exclusively used. If, for instance, in such cases a statical breaking strength of $t=3{,}870$ kilogrammes per square centimeter resulted, 3,500 perhaps, was taken as a low value; so, if not still more cautiously, similar judgment must be exercised in the choice of a. Beyond this, the help of factors of safety was always resorted to, and this must still be so in the future. In the following determinations particular reference is made to iron bridges. For simple compression, in accordance with the usual practice, the same limiting stresses as for tension are adopted.

Alternating stresses, either exclusively tensile, or exclusively compressive.—In what follows the numerical intensities of stress are expressed in kilogrammes per square centimeter. With iron for axles of the "Phoenix" Company, bending experiments of Wöhler's gave $t=4{,}020$, $u=2{,}190$; whence $\frac{t-u}{u}=\frac{5}{6}$; and by (16)

$$a=u\left(1+\frac{t-u}{u}\Phi\right)=2{,}190\left(1+\frac{5}{6}\Phi\right).$$

If the formula is to afford the desired security for any tensile or compressive stress, a start must always be made from the most unfavorable case of this stress. For the same iron, with the direct application of tensile stress, $u=2{,}190$ also; but $t=3{,}290$ only. Therefore take

$$\frac{t-u}{u}=\frac{3{,}290-2{,}190}{2{,}190}=\frac{1}{2},$$

very approximately; and suppose the ultimate working strength be further reduced by rounding off the value of u on the side of safety, to

$$a=2{,}100\left(1+\frac{1}{2}\Phi\right).$$

If $\frac{1}{3}$ is chosen as a suitable factor of safety, the admissible stress per square centimeter then becomes

$$b=700\left(1+\frac{\Phi}{2}\right) \quad . \quad . \quad . \quad . \quad (18)$$

Alternation between tensile and compressive stresses.—For the Phoenix iron above-mentioned Wöhler found $u=2{,}190$, $s=1{,}170$; hence in this case $\frac{u-s}{u}=\frac{7}{15}$, and by (17),

$$a=u\left(1+\frac{u-s}{u}\Phi\right)=2{,}190\left(1+\frac{7}{15}\Phi\right).$$

Rounding off these values on the safe side by diminishing 2,190 to 2,100 and increasing $\frac{7}{15}$ to $\frac{1}{2}$, since Φ is negative, there results

$$a=2{,}100\ (1+\tfrac{1}{2}\Phi),$$

whence, with the factor of safety $\frac{1}{3}$ the admissible stress per square centimeter becomes

$$b=700\left(1+\frac{\Phi}{2}\right) \quad . \quad . \quad . \quad (19)$$

Shearing Stresses.—In most practical cases a and b for shearing stresses may be made $\frac{4}{5}$ of what they would be with the same ratio of stress Φ for tension and compression.* Therefore, generally,

$$b=560\left(1+\frac{\Phi}{2}\right) \quad . \quad . \quad . \quad (20)$$

where Φ is positive or negative, according as the alternating limits of stress are in the same or in opposite senses.

X. REMARKS ON THE CHOICE OF b.

It has been shown in the above that the stress per square centimeter may be generally expressed,

for positive Φ by $b=v\ (1+m\Phi)$. (21)
" negative Φ " $b=v\ (1+n\ \Phi)$. (22)

where v, m, n, are constants depending on the nature of the material and sense of stress, Φ the ratio of the less to the greater limiting stress. When the admissible stress per unit of area has been fixed, the necessary net sectional area is found by

$$F=\frac{\max B}{b}.$$

For iron, in particular, as explained in IX., there may be assumed for tension and compression $v=700$, $m=n=\frac{1}{2}$, so that very simply,

$$\left.\begin{aligned} b&=700\left(1+\frac{\Phi}{2}\right),\\ F&=\frac{\max B}{700\left(1+\frac{\Phi}{2}\right)}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (23)$$

By this threefold safety is attained if

$$t=3{,}150,\ u=2{,}100,\ s=1{,}050.$$

Smaller values of strength for bridges than these figures indicate are certainly not necessary, and have not been hitherto customary. Notwithstanding this, as has already been remarked in I., these values can have no universal application, and should they be thought too high for the material and the purpose, the following may, perhaps, be suitable: $v=640$, $m=n=\frac{1}{2}$; whence

$$\left.\begin{aligned} b&=640\left(1+\frac{\Phi}{2}\right),\\ F&=\frac{\max B}{640\left(1+\frac{\Phi}{2}\right)}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (24)$$

Different values also may be assigned to m and n; for steel, for instance, different values cannot be avoided.*

Exception† has been taken to the fraction $\frac{1}{2}$. With equal justice the choice of $b=700$ might have been considered dubious, and 687 preferred. It may suffice to answer that, in these objections, general and special facts have been confused.

When for booms of girders p is the dead load, q the total load per lineal meter, and $\Phi=\frac{\min B}{\max B}=\frac{p}{q}$ then

$$b=v\left(1+m\frac{p}{q}\right)=700\left(1+\frac{p}{2q}\right); \quad . \quad . \quad (25)$$

these latter numerical values for wrought iron being according to IX.

When the numerical values assumed for b, the admissible stresses for a static load, for alternations between tension and an unloaded condition, and for alternations between tension and compression of equal amount, would stand in

* *Vide* "Strength and Determination of the dimensions of Structures of Iron and Steel," by Dr. P. J. J. Weyrauch, pp. 81-88 *ante*.

* *Vide* Weyrauch "Strength and Determination," &c., pp. 60-68.
† *Vide* "Engineering," vol. xxix., p. 263.

the ratio 3 : 2 : 1. The author is given to understand that this ratio has been adopted as a matter of experience in the practical construction of machinery, independently of Wöhler.

Launhardt quotes an older method of American engineers. According to this method, if tension alone alternating from 0 to 700 kilogrammes per square centimeter were admissible, for alternations of tension and compression the following formula was used:

$$F=\frac{\max B+\max B'}{700} \quad . \quad . \quad (26)$$

whence if $\Phi=\frac{\max B'}{\max B}$,

$$b=\frac{\max B}{F}=\frac{700}{1+\Phi} \quad . \quad . \quad (27)$$

Comparing this with (23):

For Φ . . =	0,	$\frac{1}{4}$,	$\frac{1}{2}$,	$\frac{3}{4}$,	1,
according to formula (27) b =	700,	560,	477,	400,	350
to formula (23) b =	700,	612,	525,	437,	350

Wöhler concluded from his own experiments that the following values were admissible for iron structures of unlimited duration:

For alternations between tension or compression only, and an unloaded condition, b=1,100 (above, 700), for alternations between equal tensile and compressive stresses, b=580 (above, 350).

Previous to his becoming acquainted with the Weyrauch formula, Launhardt recommended a provisional adoption of (26).

XI. ON THE LIABILITY TO BUCKLING.

In the preceding demonstration, the buckling strength has not been considered.

Other methods proposed in Germany and Austria have also left this question out of consideration,* as it was formerly almost universally the practice in those countries to calculate even the struts of lattice bridges for simple compression only. It was sought to avoid the risk of buckling by large moments of inertia of sections, and when necessary by fixing intermediate points. Since, however, the neglect of the liability to buckling had been noticed by American reviewers, the author first showed how it could be taken into account in the Italian edition of the work already frequently quoted. The following short demonstration may serve as a supplement to the English edition.

Of course it is not a question of improving the very defective theory of buckling of Euler and Navier, still less of reconstructing the empirical formulæ of Rankine and Gordon. Nevertheless, the author is so much convinced of the inaccuracy of the latter makeshifts, that it appears incredible to him how some English engineers, who remain contented with them, could raise objections on the score of inaccuracy to the formulæ given above.

According to the preceding (X), if F_d denote the requisite sectional area of a strut without liability to buckling, the stresses on which alternate between the maximum compression B_d and the minimum compression B_δ, then

$$F_d=\frac{B_d}{b}=\frac{B_d}{v\left(1+m\frac{B_\delta}{B_d}\right)}; \quad . \quad . \quad . \quad (28)$$

and for a bar, which is strained between the maximum compression B_d and the maximum tension B_t, there follows,

$$\text{for } B_d > B_t, F_d=\frac{B_d}{b}=\frac{B_d}{v\left(1-n\frac{B_t}{B_d}\right)} \quad . \quad . \quad (29)$$

$$\text{for } B_d < B_t, F_d=\frac{B_t}{b}=\frac{B_t}{v\left(1-n\frac{B_d}{B_t}\right)} \quad . \quad . \quad (30)$$

All stresses are to be substituted here as numerical values without signs.

XII. CONSIDERATION OF THE LIABILITY TO BUCKLING.

If for any strut or pillar, l be the length, F the sectional area, I the least moment of inertia of the section for axes through its center of gravity and lying in its plane, E the modulus of elasticity, t the statical breaking strength for simple compression only, δ a coefficient known from the theory of buckling depending on the method of securing the ends, let the following expression be introduced:

$$\mu=\frac{Ftl^2}{\delta EI}=\frac{Fl^2}{I\sigma} \text{ where } \sigma=\frac{\delta E}{t} \quad . \quad . \quad (31)$$

and there results, according to the

* Shäffer in his modification of Gerber's method has subsequently taken it into account. Deutsche Bauzeitung, 1877.

theory, as mean stress per unit of section at the moment of buckling:

$$t_k = \frac{\delta EI}{Fl^2} = \frac{t}{\mu} \quad . \quad . \quad (32)$$

For the transmission of a pressure B_d there is required with a static load, without liability to buckling, a section $\frac{B_d}{t}$; with liability to buckling, a section $\frac{B_d}{t_k}$. In a sectional area F, the greatest stress per unit of area is without liability to buckling $\frac{B_d}{F}$.

Let k be the hypothetical stress per unit of area obtained on the supposition that the greatest stresses implied in (28) and (32) have the same ratio as the required sectional areas. Then

$$k : \frac{B_d}{F} = \frac{B_d}{t_k} : \frac{B_d}{t}.$$

Substituting from (32) there follows:

$$k = \frac{B_d}{F} \cdot \frac{t}{t_k} = \frac{\mu B_d}{F}.$$

This stress would be that due to a force equally distributed over the whole sectional area,

$$B_k = Fk = \mu = B_d \quad . \quad . \quad (33)$$

If B_k be substituted throughout in place of B_d, the same formulæ apply as without liability to buckling. The compressive force B_d, with liability to buckling, is equivalent to a compressive force B_k without liability to buckling.

The necessary sectional area on the assumption of a liability to buckling follows from (28)–(30):
for bars under compression only

$$F = \frac{B_k}{b} = \frac{\mu B_d}{b} = \mu F_d; \quad . \quad . \quad . \quad (34)$$

for alternations of tensile and compressive loads, B_t and B_d respectively:

$$\text{when } \mu B_d > B_t, \; F = \frac{B_k}{b} = \frac{\mu B_d}{v\left(1 - \frac{nB_t}{\mu B_d}\right)} \quad (35)$$

$$\text{when } \mu B_d < B_t, F = \frac{B_t}{b} = \frac{B_t}{v\left(1 - \frac{n\mu B_d}{B_t}\right)} \quad (36)$$

XIII. ANOTHER METHOD OF CONSIDERING THE LIABILITY TO BUCKLING.

The formulæ (34)–(36) derived from Navier's theory of buckling are only to be used for $\mu > 1$, as for $\mu < 1$ the calculation for uniform compressive stress only by (28)–(30) leads to greater sections. Hitherto in determining dimensions many engineers have been in the habit of combining the formulæ for uniform compressive stress and buckling, so that instead of writing in the one case without liability to buckling $Ft = B_d$ and in the other with liability to buckling $Ft = B_k = \mu B_d$, they introduced, generally, according to the practice of Rankine and Gordon,

$$Ft = (1+\mu)B_d, \; B_d = \frac{Ft}{1+\mu} \quad . \quad . \quad . \quad (37)$$

In an exactly corresponding manner the method of determining dimensions here under discussion may be proceeded with. Substitute, instead of in the one case B_d and in the other μB_d, generally, $(1+\mu)\ B_d$; whence instead of the formulæ (28)–30) applicable only for $\mu < 1$ and of those (34)–(36) admissible only for $\mu > 1$, the following formulæ, always applicable to bars under compression, with or without liability to buckling, appear:

for bars under compressive load only:

$$F = (1+\mu)F_d \quad . \quad . \quad . \quad (38)$$

for alterations of tensile and compressive loads B_t and B_d respectively:

$$\text{if } (1+\mu)B_d > B_t, \; F = \frac{(1+\mu)B_d}{v\left(1 - \frac{nB_t}{(1+\mu)B_d}\right)} \quad . \quad . \quad . \quad . \quad (39)$$

$$\text{if } (1+\mu)B_d < B_t, \; F = \frac{B_t}{v\left(1 - \frac{n(1+\mu)B_d}{B_t}\right)} \quad . \quad . \quad . \quad . \quad (40)$$

These equations resolve themselves for $\mu = 0$ into those applicable for simple compression only (28)–(30), and for μ very large as compared with 1 into the formulæ (34)–(36) derived from theory of buckling. They give larger sections than each of the corresponding others respectively.

XIV. ORDINARY PRACTICE WITH LIABILTY TO BUCKLING.

The method pursued in determining dimensions with reference to buckling will, in most cases, be the following:

In the first place F_d is determined for simple compression only by (28)–(30), the section is provisionally arranged accordingly, μ is found by (31), and the formulæ (34)–(36) or (38)–(40) can then be applied.

The struts of bridges have usually a spread-out (*gespreizten*) section, and in most cases it will be found that $\mu<1$, so that for those who adopt Navier's buckling theory, and therefore the formulæ (34)–(36), no liability to buckling exists, and the values of F_d remain as the final sectional areas. Those, however, who have hitherto used the empirical formulæ (37) in determining dimensions, may, with equal propriety, apply the new method by means of the equations (38)–(40).

The increase of section from F_d to F, rendered necessary by the liability to buckling, may in struts of bridges be usually attained by a slight strengthening of their flanges. $\frac{I}{F}$ and $\frac{I_d}{F_d}$, where I_d is the value of I for the increased section, then differ so little that another calculation of F with the more accurate μ is superfluous, the more so as the coefficient σ is in no case very reliably determined.

This coefficient would be calculated for the various methods of securing the ends of struts with reference to the values of δ corresponding to the theory of buckling from (31). At the same time the theory does not agree particularly well with experience; and, in practice, ends absolutely rigidly fixed, or perfectly free to turn round the centers of their joints, by reason of frictionless articulation, do not actually occur. For iron bars with so-called fixed ends, the author is in the habit of making $\sigma=24{,}000$;* for iron bars with both ends secured by pins (American bridges) $\sigma=18{,}000$ has been given.†

For the other constants occurring in the preceding formulæ $v=700$, $m=n=\frac{1}{2}$.

*For this values from 10,000 to 36,000 are in use. With E=2,000,000, and l=3,300 kils. per square centimeter, the theory gives, according to (31), σ=23,900. Later American experiments (Civilingeineur, 1878, pp. 17–28) lead with the assumption of formula (37) to a mean value 22,700.

† Prof. Cain in Van Nostrand's Eclectic Eng. Mag., 1877, p. 458.

XV. EXAMPLES.

Considering the simplicity of the method of determining dimensions here recommended, few examples are needed.*

Example 1.—The stress on a bar, normal to its cross section, varies between the tension 50,000 and the tension 20,000 kilogrammes. Required the admissible stress per square centimeter and the necessary sectional area.

$$\text{By (23)}\quad b=700\left(1+\frac{1}{2}\cdot\frac{20{,}000}{50{,}000}\right)=340$$

kilogrammes per sq. centim.

$$F=\frac{50{,}000}{840}=59.52 \text{ square centimeters.}$$

Example 2.—The stress alternates, in a similar case, between the tension 40,000 and the compression 30,000 kilogs. The sectional area, without reference to buckling, is to be determined.

By (23)

$$F=\frac{40{,}000}{700\left(1-\frac{30{,}000}{2\times 40{,}000}\right)}=91.43 \text{ sq. centimeters.}$$

Where the admissible stress per square centimeter

$$b=700\left(1-\frac{30{,}000}{2\times 40{,}000}\right)=437.5 \text{ kilogrammes.}$$

Example 3.—What limiting stress per square centimeter should be chosen for the booms of a lattice girder when the ratio of the dead weight to the greatest total load amounts to $\frac{p}{q}=\frac{1}{3.5}$?

By (25)

$$b=700\left(1+\frac{1}{2\times 3.5}\right)=800 \text{ kilogrammes.}$$

With 640 instead of 700 it would be 745.

Example 4.—The load on a hollow cylindrical column varies between the compression $B_d=50{,}000$ and the compression $B_\delta=20{,}000$ kilogrammes. The sectional area for the length $l=400$ cen-

*For further explanation for special structures, see Weyrauch, "Strength and Determination, &c.;" on the calculation of rivet joints, the same work; on the consideration of the liability to buckling Weyrauch, "Stabilita dele construzioni in ferro ed in acciaio;" on a special method of taking into account the shocks produced by the moving load, Zeitschr. d. Ostreich Ing. u. Arch. Vereins, 1879.

timeters is to be calculated with reference to buckling.

From (28) or (23) or Example 1, where there is no liability to buckling, F_d = 59·52 square centimeters. Corresponding to this, the section is temporarily assumed as a ring of 10 centimeters outer, and 9 centimeters inner radius, whence follows F = 59.7, I = 2,700 centimeter-units. Equation (31) gives

$$\mu=\frac{59.7\times400^2}{2{,}700\times24{,}000}=0.15.$$

According to the theory there is therefore no liability to buckling, and the calculated section might be adhered to, since formula (34) leads to a smaller value. According to the empirical equations given in XIII., on the other hand, the choice would have to be

$$F = (1 + 0.15)\ 59.52 = 68.45$$

Square centimeters. This could be attained by keeping to the outer radius of 10 centimeters, by reducing the inner radius to 8.84, giving F = 68.6.

Example 5.—The stress on a strut in the web of a bridge girder alternates between the tension B_l = 40,000 and the compression B_d = 30,000 kilogrammes. The sectional area for the length l = 200 centimeters, with reference to buckling, is to be determined, the ends being fixed.

From (30) or (23), and as already shown in Example 2, without liability to buckling, F_d = 91·43 square centimeters. The strut is constructed in the manner of a plate girder, of four angle-irons, 8 + 8 + 1 centimeters, and a web 32 + 1 centimeters; whence F = 92 and the smallest moment of inertia of the cross section, I = 852 centimeter units. Formula (31) gives

$$\mu=\frac{92\times200^2}{852\times24{,}000}=0.18,$$

so that here, too, by the theory there is no liability to buckling. If, however, the empirical formulæ are to be used, then from (40), since $(1+\mu)\ B_d = 35400 < B_l$.

$$F=\frac{40{,}000}{700\left(1-\frac{35{,}400}{2\times40{,}000}\right)}=182.5 \text{ sq. centim}$$

To make up this section the thickness of the angle irons may be assumed as 1.2 centimeter, and then F = 103.0 centimeters.

If the calculation were repeated with the more exact values of F and I, there would result consecutively I = 1028, μ = 0.17, F = 101.82. Consequently a value of F has been determined to small by 0.68 square centimeter = $\frac{2}{3}$ per cent. an error that may be neglected.

Example 6.—To calculate the strut given in the preceding example for the length l = 300 centimeters.

In this case the provisional values of F and I are exactly as above; on the other hand μ = 0.40, and since $(1+\mu)\ B_d = 42{,}000 > B_l$, by (39)

$$F=\frac{42{,}000}{700\left(1-\frac{40{,}000}{2\times42{,}000}\right)}=114.55 \text{ sq. centim.}$$

XVI. Conclusion,

It is possible, as already indicated, to find many deficiencies in the method demonstrated. But, there are here only two questions for consideration; (1) Does the method lead to the expectation of greater security than that hitherto used in determining dimensions? (2) Is not another newer method to be preferred? In order to facilitate in the latter particular the judgment and independent choice of his English professional fellow-workers the author will be ready to give in a second Paper, a short explanation and comparison of all the methods hitherto proposed. His object is mainly to supersede the present method of determining dimensions. Even if it is thought that other formulæ for a and b should be preferred, the general principles here demonstrated may yet have contributed to the elucidation of the question.

Since the strength of materials has recently acquired a fresh interest, it may well be that at no distant time satisfactory relations that will admit of general application may be discovered. The author would welcome them with pleasure. It is sufficient for him to have contributed towards the supercession of the older method of determining dimensions so as to make way for a more rational system.

THE AMOUNT OF RAINFALL IN ITS RELATION TO THE WATER SUPPLY OF A CITY.

By JULIUS W. ADAMS.

Written for VAN NOSTRAND'S ENGINEERING MAGAZINE.

AN article under the above heading appeared in the last number of the Magazine, upon which I would beg leave in the interests of the younger members of the profession, who consult your pages with profit, to make a few brief comments.

The true theory of any subject, must, of course, embrace all the facts known of it. Thus far we have too few facts upon which to build a comprehensive system—not to say—science of meteorology, and some time will elapse before we can pretend to reduce to rule the many anomolous phenomena attending the rain fall, and the resulting drainage from the water shed.

The writer of the article in question, after a review of the rain-producing processes over varied surfaces of the earth, and enumerating certain general principles which are presumed to affect the precipitation of moisture, takes the average annual rain fall in sections of the United States, and dividing these sections into nine groups, obtains what is characterized as a *type curve* for each group, representing the fluctuations of the monthly rain fall; and derives from the average rain fall of a series of years the factor 0.8, as representing the ratio of the rain fall for the dry years. Taking a twelfth of the annual rain fall for the monthly mean, he determines from the record of averages of the Atlantic coast stream flows, the ratio of the monthly flow of the streams to this mean as unity, and upon the *supposition* that 50 per cent. of the annual rain fall is collected in the streams, and 0.8 represents the ratio of a series of dry years to the mean annual rain, obtains a mean monthly rain fall, which combined with the monthly ratio of the flow of the streams above noted, gives the available flow in inches monthly, and adds—"which quantities have only to be multiplied by the number of acres or square miles in the water shed to determine the amount available." Subsequently, making an allowance of 60 per cent. for the annual evaporation from reservoir surfaces, and obtaining a monthly mean and ratio for this, finally embodies the whole in a table for use, "in the calculations upon water-works."

If the writer had confined himself to the phenomena attending the local rain fall, the deductions drawn would have been interesting to the extent of showing what had taken place, and what might be anticipated in the locality chosen—Troy for instance; but as he does not state explicitly, that his deductions are to be taken with great caution when used in estimating a city water supply, but if I understand the scope of the article, it is, that it offers a method by which, at least in the Atlantic States, and to the extent of 100 square miles of shed, the amount of water derivable from an estimate of the *mean annual* rains, may be used safely in estimating the water supply of cities. It is this which I take exception to—the application of any general formula for such purpose. However closely the results of using this method may approximate the truth in particular cases, as shown by existing works already established, and that it may do so is undoubted, yet it cannot be assumed, *a priori*, as applicable in the future for other localities. The whole principle of estimating from yearly averages of rain fall has been proven by experience to be misleading, and, for the purposes of a permanent city supply, no longer recommended.

Much misapprehension exists as to the amount of water which may be calculated upon with certainty on a given water shed; and disappointment in many notable cases, which might be cited, has arisen from taking as a standard for computation the average rainfall of a series

of years. Even when a compensation is aimed at by large storage facilities, it is found to be impracticable to retain all the *average* counted on; as in the event of large floods occurring when the reservoirs are full, the water must be lost, and the available supply, obtainable from reservoirs, is found to fall short of that deduced from computations based upon average rainfalls. A single statement from the official report of the engineer of the Liverpool works will illustrate this. The average yearly fall for thirty years' observation showed 48 inches, and allowing 50 per cent. as collectable, 24 inches was assumed as available; with $3\frac{3}{4}$ million gallons per acre of the shed as storage room, 18 million gallons daily was estimated as the capacity of the works. (This was in addition to compensation water to certain mills.) Three dry years in succession reduced the delivery to six million gallons daily, or $9\frac{3}{4}$ inches instead of 24 inches over the entire shed. This case is but one of many bearing the same evidence.

Before a true average of rainfall can be determined upon in a given locality, of course the proper proportion of *dry* years should be embraced in the series, otherwise the estimate will be in excess —aside from the fact, that unless under exceptional circumstances, the minimum years of rainfall, and not the average become the true criterion for certainty of supply. And in addition to this, the year of least rain is not necessarily the year in which the season of least flow from the streams occur, and this latter is the measure of reliable supply. Nor is the year of least rainfall necessarily the year of drought, although frequently so; but the year of drought which tests the capacity of a city supply is the year wherein the rainfall is so unequally distributed, that for several months in succession, it may be, the evaporation and absorption and needs of vegetable life, take up all the rain which falls, and thus little or nothing is contributed to the city supply.

On the subject of evaporation from reservoir surfaces, it is manifestly an error to assume it as a percentage of the rainfall; for beyond the fact that during rains but little evaporation may be taking place, and at such times there may be a lowering of the temperature, which would tend to retard evaporation, there is nothing in a season of rain influencing to any extent such evaporation. Exposure to winds, hygrometric conditions of the atmosphere, the vicinity of forests or open plains, the presence or absence of elevated surroundings—all may effect it more or less, but independently of rainfall, and no connection necessarily exists between the two. The proper determination of the amount of evaporation from water surface is so replete with difficulties, and some of the results attained are thus far so anomalous, that the effort is no longer made to attain absolute accuracy in this respect; and it is assumed, in accordance with some observations, that the yearly rainfall on a given water surface, will supplement the loss by evaporation; hence, in considering the area of a gathering ground, if the water surfaces are subtracted in estimating the area of the shed, and disregarding the element of evaporation from these surfaces, it will be as near the truth as can be reached by any known method of observing evaporation. And so of the absorption of the soil, the demands of vegetation, and evaporation from ground surfaces, which are usually embraced under one head, can, neither collectively nor singly, bear any relation whatever to the amount of rainfall, and can only be approximately determined by long-continued observation and measurement on areas of similar characteristics, and the result deducted from the yearly rain, and independently of the extent of the latter. Neither of these items, any more than that of the evaporation from water surface, can be properly estimated as a percentage of the rainfall.

In the article referred to, reference is constantly made to the average annual rainfall as a basis for computation, and the low years precipitation is derived from it by the simple process of multiplying by 0.8; but unless the observations had extended through a greater number of years than is usual with such records in this country, and had certainly embraced a year or more of the *lowest* rainfall, or the season of lowest flow, it would scarcely prove a safe guide.

The flow from the Merimack, the Passaic, the Delaware, the Schuylkill and the Croton, as well as others of our

rivers, may be computed on an average at one million of gallons daily throughout the year for every square mile of drainage area. But of what use is the knowledge of this fact, when it is equally certain, that save by an amount of storage room which is wholly impracticable, the freshet flows, which supplementing the low summer flow brings up the total to the average named, cannot be retained.

It is estimated that to avail of the average yearly flow of the Croton basin of 338 square miles, would require a storage of 41000 million gallons. This would accomplish it upon paper, but for obvious reasons, the reservoirs could not be counted on at all times to fulfill their purpose, were it practicable to build them; and hence a less discharge from the basin becomes the true measure of its capacity.

From 1864 to the present year, the average yearly rainfall on the Croton basin has been 46.1 inches. In the year 1877 the rainfall was so nearly the same (46.03) that the results of that year's observation are taken to represent the average. There was collected that year in the Croton, 45%, or 21 inches, as available rain, and the daily average flow for the year was a million gallons per square mile. Of the rain for that year, 15.8% was stored or used in the City of New York, 30.7% was lost for lack of storage capacity, and 53½% was evaporated from the surface of the shed (omitting the water surface) or absorbed, or used for vegetable life and did not reach the springs.

The rainfall the last year (1880) on the Croton shed was 38.5 inches; and the following represents the ratio of the several items of water supply for the several years named, on this basin and Ridgwood, L. I. In Blodgett's Climatology it appears that the Croton basin is credited with an average annual rain of 44 inches, and the Long Island basin with 42 inches. In the last 50 years the observations of the rain gauges have shown an average of 42.69 inches yearly rain, for the water-shed which supplies the City of Brooklyn.

	Years.	Square miles watershed.	Inches of rainfall.	Percentages of yearly rainfall.			
				Gathered in the streams.	Evaporated, absorbed and lost to the City supply.	Gathered, but lost for lack of storage.	Stored and used in the City.
Croton.....	1877	338	46.03	46.5	53.5	30.7	15.8
	1880	388	38.5	39.	61.	24.5	14.5
Brooklyn ..	1880	60	38.58	25.27	74.73	nothing.	25.27

It appears by the above, as might be anticipated, that the loss by evaporation and absorption, instead of being proportional in any degree directly to the amount of rainfall, is really inversely proportional thereto; and indeed an acquaintance with the detailed operations of almost any well-conducted city water supply, will show how little the *average* rainfall can be trusted as a safe guide, and the necessity which exists of referring, as of controlling importance, to those local characteristics, topographical and geological, as well as meteorological, which will be found to modify most materially the results arrived at by any general formula.

In a dry season the Croton basin has yielded a minimum of 100,000 gallons per square mile in twenty-four hours, and during the same season the Brooklyn basin has yielded a minimum of 300,000 gallons per square mile for the same time, the rainfall for the year being in favor of the Croton basin. This discrepancy in the amounts flowing from a given area, appearing within so short a distance as the width of the East River, will indicate to what extent local circumstances may influence the water supply of a city, independently of the extent of rainfall. Much has been written abroad upon this subject of water supply, but scarcely applicable to this latitude.

ON THE VARIOUS MODES OF TRANSMISSION OF POWER TO A DISTANCE.

By M. A. ACHARD.

A Paper read before the Institution of Mechanical Engineers.

THE author in this paper furnished a summary of the practical results obtained in the transmission of power to a distance. While the interest attaching to this subject is unquestionable, the author is, nevertheless, very doubtful whether a successful result can be attained in one particular application—namely, the establishment of large undertakings for distributing hydraulic power to a number of factories, either existing or contemplated, similar to the undertakings at Schaffhausen, Fribourg, and Bellegarde. At the first of these places, in spite of favorable circumstances, rapid extension of working, and good management, the profit has been very small on the capital outlay. The manufactories at the two other places, being much less favorably situated, have failed after a short and profitless existence. Their failure has shown very clearly that their founders labored under a strange delusion in supposing that cheap motive power was in itself sufficient to create industries in localities where their essential elements were wanting. The author accordingly considers there is not much to be gained from this method of transmitting power to a distance, and that it can only succeed financially under exceptionally favorable conditions. Having promised so much, he next proceeded to examine the various methods used, or proposed, for transmitting power to a distance. He first considered transmission of power by wire ropes, which is merely an extension of the simple case of transmission by ordinary hemp ropes, and the same principles apply to both. Let A and B be the axes of two parallel shafts carrying two pulleys whose planes coincide. The driving power P acts on A, and the resistance Q on B. For simplicity, let it be assumed that those two forces act tangentially at the circumference of the pulleys. The motion is communicated from A to B by means of the rope passing round the two pulleys; of this the part which is passing towards the driving pulley is called the driving span, and the part which is passing from the driving pulley is called the trailing span. Let T be the tension of the driving span, and t that of the trailing span. Neglecting friction, &c., we should have Q=P; and the values of the tensions in the two spans are given by the equations

$$T-t=P \text{ and } \frac{T}{t}=k;$$

denoting by k the smallest practicable value of e^{fa} for the two pulleys, where e is the base of Napierian logarithms, f the coefficient of friction between the pullev and the rope, and a the ratio between the arc encircled by the rope and the radius of the pulley. Accordingly the values of T and t are given by the following equations:—

$$T=\frac{k}{k-1}P;\ t=\frac{P}{k-1}.$$

If the ratio $\frac{T}{t}$ be greater than k, the rope will slip on the driving pulley. The values of T and t, as above calculated, when k has its exact value, are only just sufficient to prevent slipping, which would occur on any accidental diminution of friction. For safety, therefore, it is necessary to assign to k a somewhat lower value than its real one; which practically amounts to increasing the tensions T and t a little beyond what is requisite in theory. The tension common to the whole rope when at rest is somewhere intermediate between the tensions T and t of its two spans while running; and by adjusting the rope while at rest to this intermediate tension, its two spans assume of their own accord the required tensions T and t as soon as it begins to run. The section w to be given to the rope, so that it may possess the requisite strength, is regulated by

the driving tension T, and must be such that the quotient $\frac{T}{w}$ shall not exceed the working strain which the material of the rope is suited to bear in practice. It is evident that, in transmitting a given amount of power, the driving tension, and consequently the section of the rope, may be diminished by increasing the speed; for if N denotes the power transmitted, and v the speed of the rope, then

$$Pv=N, \text{ and } T=\frac{kP}{k-1}=\frac{k}{k-1}\ \frac{N}{v}.$$

In practice the rope elongates under the continuous pull, and requires shortening from time to time to keep the tension up to the proper amount. The author next took into account the useless resistances neglected for the sake of simplicity. The useful resistance Q is now necessarily less than the driving power P, and the ratio $\frac{Q}{P}$ represents the efficiency of the transmission. The useless resistances are two in number. The first is the rigidity or stiffness, due to the imperfect flexibility of the rope. This effect, however, is insignificant in the case of rope transmission, on account of the large size of the pulleys employed. The other useless resistance is the friction of the two shafts A and B in their bearings, which is measured by the resultant F of all the external forces acting on each shaft. It appears from the principles enunciated above that the employment of rope transmission renders this friction considerable. In fact, under average conditions of adhesion, the value to be allowed for $k=e^{fa}$ is not more than 2; and since in the limit

$$\frac{T}{t}=k \text{ and } T=\frac{kP}{k-1},$$

we have as the least possible values T=2 P, and t=P. These tensions are parallel to each other, and as the useful resistance Q may also act in the same direction, the total pressure F on the shaft may=T+t+P=4 P, as a minimum, where the conditions are the most unfavorable; while under the most favorable conditions the pressure on the bearings will be given by F=T+t−P=2 P, as a minimum. Hence the average pressure may be taken as at least 3 P. It is evident, therefore, that rope transmission renders the shaft friction much greater than does transmission by toothed wheels. But the effect of this friction is much reduced by the large diameter of the pulleys in comparison with that of their shafts, in consequence of which the pressure on the shaft bearings, has to be multiplied by a number not exceeding at most 0.003, in order to obtain the resulting tension on the rope. The author dealt briefly with belting and went on to consider at length the nature of the strains to which ropes are subject. It is difficult to lay down any general rule as to the duration of the ropes, for this depends upon the conditions under which they work. In practice, it must not be assumed that a rope in constant use will last more than a year. In fact, Professor Amsler-Laffon recently wrote to the author on the subject of the ropes at Schaffhausen:—"A rope lasts about one year, some a little more, some a little less. But it must be understood that we do not wait till our ropes break, but replace them as soon as we can no longer depend on their strength. They might therefore last rather longer, if we chose to run the risk of interruption in our work." Their short life is certainly a defect in this mode of transmitting power. According to M. Ziegler, who has considerable experience on this subject, horizontal oscillations are very injurious to the duration of the ropes, and they appear to last longer on pulleys with wide grooves than with narrow grooves. The author next considered transmission by compressed air. Hitherto the method of transmission by compressed air has only been used, so far as the author is aware, for boring the headings of mines, and the long tunnels through the Alps. In these cases, as is well known, the work to be done consists in a rapid boring of holes for the purpose of blasting the rock with powder or dynamite. As this kind of work requires a high pressure of air, and almost entirely precludes the employment of expansion, the utilization of the motive force is necessarily defective; but in consequence of the peculiar convenience which compressed air offers for the work, and particularly the improved ventilation which it affords, the advantage of its employment is un-

doubted, and leaves in the background the question of efficiency. M. Achard dealt at great length with the somewhat complex mathematics of the subject, but he did not supply much if any new data. Referring to the motors fed with the compressed air, the author held that this subject is still in its infancy from a practical point of view. In proportion as the air becomes hot by compression, so it cools by expansion, if the vessel containing it is impermeable to heat. Under these conditions, it gives out, in expanding, a power appreciably less than if it retained its original temperature, besides which the fall of temperature may impede the working of the machine, by freezing the vapor of water contained in the air. If it is desired to utilize to the utmost the force stored up in the compressed air, it is necessary to endeavor to supply heat to the air during expansion, so as to keep its temperature constant. It would be possible to attain this object by the same means which prevent heating from compression, namely, by the circulation and injection of water. It would, perhaps, be necessary to employ a little larger quantity of water for injection, as the water, instead of acting by virtue both of its heat of vaporization and of its specific heat, can in this case act only by virtue of the latter. These methods might be employed without difficulty for air machines of some size. It would be more difficult to apply them to small household machines, in which simplicity is an essential element; and we must rest satisfied with imperfect methods, such as proximity to a stove, or the immersion of a cylinder in a tank of water. Consequently, loss of power by cooling and by incomplete expansion cannot be avoided. The only way to diminish the relative amount of this loss is to employ compressed air at a pressure not exceeding three or four atmospheres. The only real practical advance made in this matter is M. Mekarski's compressed air engine for tramways. In this engine the air is made to pass through a small boiler, containing water at a temperature of about 120 deg. Cent.—248 deg. Fah.—before entering the cylinder of the engine. It must be observed that in order to reduce the size of the reservoirs, which are carried on the locomotive, the air inside them must be very highly compressed, and that in going from the reservoir into the cylinder it passes through a reducing valve, or expander, which keeps the pressure of admission at a definite figure, so that the locomotive can continue working so long as the supply of air contained in the reservoir has not come down to this limiting pressure. The air does not pass the expander until after it has gone through the boiler already mentioned. Therefore, if the temperature which it assumes in the boiler is 100 deg. Cent.—212 deg. Fah.—and if the limiting pressure is five atmospheres, the gas which enters the engine will be a mixture of air and water vapor at 100 deg. Cent.; and of its total pressure the vapor of water will contribute one atmosphere, and the air four atmospheres. Thus this contrivance, by a small expenditure of fuel, enables the air to act expansively without injurious cooling, and even reduces the consumption of compressed air, to an extent which compensates for part of the loss of power arising from the preliminary expansion which the air experiences, before its admission into the engine. This scheme was then mathematically investigated by the author. Next M. Achard dealt with the transmission of power by water pressure. Of machines worked by water pressure, the author referred only to two, which appear to him in every respect the most practical and advantageous. One is the well-known piston machine of M. Albert Schmid, engineer, Zurich. The cylinder is oscillating, and the distribution is effected, without an eccentric, by the relative motion of two curved surfaces fitted one against the other, and having the axis of oscillation for a common axis. The convex surface, which is movable and forms part of the cylinder, serves as a port face, and has two ports in it communicating with the two ends of the cylinder. The concave surface, which is fixed and plays the part of a slide valve, contains three openings, the two outer ones serving to admit the pressure water, and the middle one to discharge the water after it has exerted its pressure. The piston has no packing; its surface of contact has two circumferential grooves, which produce a sort of water packing acting by adhésion. A small air chamber is connected with

the inlet pipe, and serves to deaden the shocks. This engine is often made with two cylinders, having their cranks at right angles. Its efficiency is equal to from 37 to 83 per cent. The other engine, which is much less used, is a turbine on Girard's system, with a horizontal axis and partial admission, exactly resembling in minature those which work in the hydraulic factory of St. Maur, near Paris. The water is introduced by means of a distributor, which is fitted in the interior of the turbine chamber, and occupies a certain portion of its circumference. This turbine has a lower efficiency than Schmid's machine, and is less suitable for high pressures; but it possesses this advantage over it—that by regulating the amount of opening of the distributor, and consequently the quantity of water admitted, the force can be altered without altering the velocity of rotation. Its efficiency varies between 35 and 68 per cent. The transmission of power by electricity was considered last of all. The author stated nothing new on this subject. He gave a table by Mr. Hagenbach, which promised to be useful, but he added that the brake measurements obtained were inaccurate, consequently the table is of little value. He concluded his paper with a retrospective glance at the four methods of transmission of power which had been examined. It would appear that transmission by ropes forms a class by itself, whilst the three other methods combine into a natural group, because they possess a character in common of the greatest importance. It may be said that all three involve a temporary transformation of the mechanical power to be utilized into potential energy. Also in each of these methods the efficiency of transmission is the product of three factors or partial efficiencies which correspond exactly—namely, (1) the efficiency of the instrument which converts the actual energy of the prime mover into potential energy; (2) the efficiency of the instrument which reconverts this potential energy into actual energy, that is into motion, and delivers it up in this shape for the actual operations which accomplish industrial work; (3) the efficiency of the intermediate agency which serves for the conveyance of potential energy from the first instrument to the second. This last factor for transmission by electricity is the exact correlative of the efficiency of the pipe in the case of compressed air, or of water pressure. It is as useful in the case of electric transmission, as of any other method, to be able, in studying the system, to estimate beforehand what results it is able to furnish; and for this purpose it is necessary to calculate exactly the factors which compose the efficiency. In order to obtain this desirable knowledge, the author considers that the three following points should form the aim of experimentalists: (1) The determination of the efficiency K of the principal kinds of magneto-electric, or dynamo-electric machines working as generators. (2) The determination of the efficiency K, of the same machines working as motors. (3) The determination of the law according to which the magnetism of the cores of these machines varies with the intensity of the current. The author added that he would gladly have concluded this paper with a comparison of the efficiencies of the four systems which have been examined, or, what amounts to the same thing, with a comparison of the losses of power which they occasion. Unfortunately such a comparison has never been made experimentally, because hitherto the opportunity of doing it in a demonstrative manner has been wanting; for the transmission of power to a distance belongs rather to the future than to the present time. The author believes that transmission by ropes furnishes the highest proportion of useful work; but that as regards a wide distribution of the transmitted power the other two methods, by air and water, might merit a preference.

The discussion which followed, the author being absent, was opened by Mr. J. N. Shoolbred, who gave facts and set forth certain experiments on the efficiency of electric generators, quoted in the paper, in a form more in accordance with English ideas. He observed that they confirmed the previous experiments of Dr. Hopkinson and others as to the very high efficiency of such generators, but this of course proved nothing as to the efficiency of transmission by electricity, where the electricity, after being generated, had to be reconverted into

power at the other end. In this reconversion Dr. Siemens' experiments showed a loss of about 50 per cent. Recent experiments of his own, however, led to the hope that this loss might be largely reduced by adopting proper precautions between the two machines, the generator and the motor. Anyhow, there were many ases where efficiency was of the first importance, *e g.*, where natural sources were utilized, or a large central steam engine could be drawn upon, &c., and here the greater compactness and economy of the electric transmission would give it important advantages. Mr. Alexander Siemens followed in the same strain, and thought far too much was made of the question of efficiency, which was often of little importance. He instanced Sir William Armstrong's arrangement at Rothbury, and that of Dr. Siemens at Tunbridge; and looked forward to the time when large steam engines would be established at central positions in towns, whence the power would be distributed by electricity as required. He held that a few experiments were necessary to settle the arrangements of such transmissions, because the conditions might always be preserved constant, by arranging the proper resistance in the leading wire; and considered that prejudice was now the only thing that hampered the transmission of power by electric means.

Mr. Fernie recalled the attention of the meeting from electricity, and gave some examples of the extensive use of transmitted water power in Switzerland. He mentioned that in Geneva the men who cut firewood for house use used small machines, which they worked by attaching them to high-pressure mains laid along the streets. M. Schönheyder then drew attention to a statement in the paper with regard to the wide belts used in America—viz., that at high speeds a partial vacuum is formed between the belt and the pulley, which gives an adhesion more like that of an ordinary leather sucker, and greatly increases the tension which the belt will bear without slipping. M. Schönheyder altogether disbelieved in the existence of this action—and probably with justice—though it has undoubtedly been claimed in the case of American belts. He discussed the reason why the iron ropes used in the transmission wore so rapidly, lasting, *e.g.*, at Schaffhausen, only twelve months; and suggested that they must either be drawn too tight, or given too high a working strain. He also spoke of the great friction of water in pipes under heavy pressure, examples of which were given in the paper; and suggested that further experiments were desirable on this subject. Mr. W. E. Rich took up a point neglected in the paper, viz., the proper pressure to be used in transmission by compressed air. This, he held, should be as low as was convenient, say, 30 lbs. per square inch, for dry air compressors. Air compression he considered was better suited to intermittent than constant transmission. In the latter case the freezing at the exhaust ports was a great inconvenience, but this might be remedied by placing a small fire near them, or otherwise. With regard to water engines, the greatest inconvenience was the slide valve, which always gave trouble, and should be done away with if possible, a result already attained in Mr. H. Davey's water engines, described by him to the Institution in 1880.

REPORTS OF ENGINEERING SOCIETIES.

THE AMERICAN SOCIETY OF CIVIL ENGINEERS.—The Society have of late engaged in the discussion of the question of wind pressure on Bridges. Paper No 213, by C. Shaler Smith, gave some interesting and valuable observations bearing on this question.

The following is an abstract from it:

For a number of years past, whenever practicable, I have personally visited the tracks of destructive storms as soon as possible after their occurrence, for the purpose of determining the maximun force and width of the path in each case.

The most violent on my records are as follows:

First.—East St. Louis, 1871: Locomotive overturned; maximum force required, 93 lbs. per square foot.

Second.—St. Charles, 1877: Jail destroyed; force required, 84.3 lbs. per square foot.

Third.—Marshfield, Mo., 1880: Brick mansion house leveled; force required, 58 lbs. per square foot.

Fourth.—Havre de Grace, Md., 1866: Ten spans wooden Howe truss bridge, 250 feet each, blown over; force required, 27 lbs. per square foot.

Fifth.—Decatur, Alabama, 1870: Two spans of Combination Triangular truss blown over; force required, 26 lbs. per square foot.

Sixth.—Meredosia, Ill., 1880: One span wooden Howe truss, 150 feet long, overturned; force, 24 lbs. per foot.

Seventh.—Omaha, Nebraska, 1877: Two spans iron Post truss, 250 feet each, blown down; force required, $18\frac{7}{16}$ lbs. per square foot.

Also, sundry cases of train derailment caused by wind, the maximum force required being $30\frac{1}{2}$ lbs. per squre foot.

In each of the foregoing cases I have given what appeared to be the maximum effort of the wind and the lowest pressure required to produce the observed result. It is therefore not unlikely that the real force of the wind in each example was greater than I have given it. Some of the tornadoes were very destructive—the Marshfield one, for instance, having cut a swath 46 miles long and 1800 feet wide, and killed and wounded over 250 people. To the above cited instances may be added the Tay Bridge disaster, in which case $20\frac{4}{16}$ lbs. per square foot on train and bridge were required to destroy the piers, through the rupture of the vertical bracing in the four bottom tiers of the pier, over which the train was passing when failure began. My reasons for considering 30 lbs. per square foot sufficient for a working specification, when the above record shows much higher pressures, are these—

First.—I very much doubt if a *direct* wind or gale ever exceeds 30 lbs. per foot; whirlwinds do exceed it, but the width of the pathway of maximum effort in these is usually very narrow, although the general direction is so erratic that the appearance of the *debris* is generally such as to produce the impression that the vortex was much larger than was really the case. With the exception of the Marshfield tornado, I have yet to find a storm swath where the width of pathway, wherein the force exceeded 30 lbs. per square foot, was more that 60 feet wide.

The St. Charles tornado is a case in point. This whirlwind cut a swath about 1000 feet wide for 14 miles, and destroyed over 300 houses, exerting a force of over 84 lbs. per square foot at its point of maximum effort. It crossed the middle span of the St. Charles bridge nearly at right angles, and developed a pressure of $52\frac{4}{16}$ lbs. per square foot in picking up and crushing a barrel of tar, which stood on the bridge in the path of the vortex. The width of the vortex was distinctly marked on the span by the circle in which the tar was spun around, the wreckage left upon it, and the points at which it ceased to destroy the flooring. This width was thus shown to be slightly over 60 feet, and, guided by this, I was subsequently enabled to locate the path traveled by the central vortex throughout the entire length of the storm swath. The bridge itself was uninjured, although it was only proportioned to withstand 30 lbs. per square foot, with a strain of 20,000 lbs. per square inch on the braces. This span was 320 feet long, 30 feet in depth, and the top chord was 120 feet above the water. I consider it very unlikely that a bridge of over 200 feet span will ever be exposed to a wind force of more than 30 lbs. per square foot, acting *in the same direction* over its entire length.

Next.—A fully loaded passenger train, and the heaviest possible freight train will leave the track at the respective pressures of $31\frac{1}{4}$ and $56\frac{1}{2}$ lbs. per square foot. If the braces are proportioned at 15,000 lbs. per square inch, with a wind pressure of 30 lbs. per square foot, they will still be within their limit of elasticity at the moment when the train is blown from the track in either case. Destruction of the span will then take place, if at all, from the effects of derailment; to resist which greater strength in the wind bracing will be of no value.

Next.—If there is no tension in the pier columns until 30 lbs. wind pressure is reached, and these columns are properly spliced and anchored, as per specifications, there will be an ample margin of tensile strength in any case where this pressure may be exceeded.

Last.—In view of the comparative rarity of these extreme strains and the consequent slight fatigue to which the iron is exposed, the high stresses imposed on the wind bracing are perfectly legitimate.

Boston Society of Civil Engineers.—At the regular February meeting, a paper on "The Fall River Bridge," prepared by the late E. N. Winslow, was read by the Secretary.

The committee to whom was referred the subject of issuing a work on the "Law and Practice for Land Surveyors in the New England States," reported adversely.

At the February meeting a paper on "Railroad Signals" was read by George W. Blodgett.

The committee on the Metric System reported on the information obtained relative to the progress toward introduction of the system into this country and the world at large.

Polytechinic Club of the American Institute.—The paper of the evening, February 24, was upon the subject of scientific books, by Mr. Wm. H. Farrington, of the house of D. Van Nostrand. It was an elaborate discussion of the subject, and was full of the most interesting information.

The noticeable feature of modern scientific literature is the tendency to give science an every-day dress, and to describe many of the most ordinary vocations from a scientific standpoint.

At the present time at least ten divisions are needed in order to reasonably classify scientific books. The last of them, the so-called "Popular Science," attempts to draw into itself all the forms and phases of modern scientific literature.

In all languages, especially English, French and German, works bearing upon construction take prominent places, and in this term "construction" it is intended to embrace steam engineering, railway science, building, &c. The engineering books easily take the foremost rank among the works of science.

One of the earliest leading publishers of scientific books, and one who in his day made this class of literature a representative one,

was John Weale, of London. We all know, I presume, what the "Weale's Series" are—the little green or red covered books, in limp or flexible cloth, with flush cut edges. Mr. Weale has been dead many years, but his series of reasonably-priced scientific treatises upon all manner of subjects still lives. It is true the advancements science has made has rendered necessary many changes in these little books; some have been rewritten and enlarged and altered as they have passed down through different publishers' hands; their number has been added to, until there are now about 200 of them, and they have had a sale of many thousands and continue to sell. To a considerable extent engineering science is predominant in this series, but they cover a wide range of other subjects. No attempt has been made to emulate this series of Weale, unless we might mention in connection with it the "Roret Manuals" and the more recent "Actualities Scientifiques," and the "Science Series," of Van Nostrand, New York, begun a few years ago.

The first named of these is—or, rather, was—a very popular series in the French language, of compact little volumes upon technology, or the science of the workshop, and were largely embraced within the domain of applied chemistry. The "Actualities Scientifiques" of Mallet-Bachelier, Paris, are no more popular than the "Weale's Series," but cover a rather wider range of subjects, and are strictly less scientific, while for Van Nostrand's we can only say, that so far as the higher branches of science are concerned, they would take precedence of all, but as individual treatises are much less exhaustive of the subject, and in many instances partake more of the character of a monograph on some special branch. Some of Mr. Weale's other publications, such as his collections of plans and drawings of bridges, with descriptive text, his large and exhaustive works upon the "Steam Engine" and on "Railways," his fine volumes on "Architecture," his "Quarterly Papers on Engineering," and another series in the same style on "Architecture," while expensive as books, are remarkable examples of scientific literature, and remain as monuments to the man John Weale more enduring than brass or stone

The relatively high price of scientific works was spoken of at length, and the reasons for it pointed out. These were the comparatively small editions which can be sold, and the greater expense entailed in the original preparation of the work. Tabulated work, algebraic formula, with cuts and diagrams, largely increase the expenses of production. In England, publishers of scientific books do not, as a rule, electrotype their works, preferring, when an edition is exhausted, to prepare a revised edition. In this country electrotyping is the rule, hence different editions are brought out with only trifling variations.

It is a noticeable fact that Rankine's works, though of the highest character—one might almost say abstruse—have had a large and steady sale in this country. This is a fact that, to say the least, speaks well for the ability of our students.

There are constant calls for books upon subjects which are treated of in the larger and more expensive technical cyclopedias. That these demands are not supplied, is largely due to the fact that in many cases the works would of necessity be filled with trade secrets.

Some of the notable scientific books were described and their curious histories given. Among others the growth of Weisbach's "Mechanics" was traced, and its curious continual publication in parts in Germany mentioned. In conclusion, Mr. Farrington said:

"Regarding the publishing houses of scientific books, I would state that in England two only confine themselves exclusively to this line—E. & F. N. Spon, and Crosby Lockwood & Co. Many other houses, as in this country, publish various works in science of all kinds, and are heavy publishers as well, such as Longmans & Co., London, or Appleton & Co., here; but Spon and Lockwood confine themselves now to science only, and the former house deals mostly in engineering and the allied sciences. Lockwoods are now the publishers of Weale's series and many other important and elaborate works. In Paris, Mallet-Bachelier, Lacroix and Dunod are the leading houses, the former, successor to Gautier-Villars, deal mostly in mathematical books, and the latter mainly in engineering, architectural and hydraulic works. Recently they have issued an exhaustive series on engineering in all branches. Dubauve's "Manuel de l'Ingénieur" in 20 octavo volumes and 12 quarto atlases of plates. Dumain of Paris confines himself to military literature exclusively. In this country the leading publishers in science alone are H. C. Baird & Co., of Philadelphia, and Van Nostrand and John Wiley & Sons, of this city. The former house publishes books of an exclusively practical nature, or, as they term it, "Industrial Science." Van Nostrand, on the other hand, has more of the theoretical class on his list, and also combines with these military and naval science. The Messrs. Wiley have never confined themselves solely to science, They are possibly doing so more of late, but still they deal in some miscellaneous literature, as well as in biblical.

Large numbers of publishers, however, throughout the country are turning their attention to works in science, and the number of books issued are increasing year by year. The proportion of scientific works published in this country, however, to the whole number in other branches of literature, is relatively small, and, in comparison to what the Germans and French are giving to the world, would seem quite insignificant. In closing these remarks, I would like to say that now that the apprentice system has been virtually abandoned, young men, who are learning a trade, have to depend largely upon their own studiousness to become proficient in their line of work. To this end these books, of which I have been speaking, are the essential helps, and I really think that, with the array of volumes presented, little excuse exists for any conscientious workman to remain in ignorance of the requirements of the trade of his choice, or fail to attain a fair proficiency in it.

ENGINEERING NOTES

ON A ROPE TRAMWAY AT STRASBURG.—(From abstracts of Inst. of Civil Engineers) —This tramway is in use at Strasburg for removing the earth from a ditch, 22 feet deep, forming part of the new system of fortifications. Starting from the lowest point of the excavation, it is carried first through a short tunnel at a rise of 1 in 10, then passes over a railway (which is protected by a temporary roofing) and thence is led horizontally to its farther or discharging end, the total length being about 1,000 yards. A peculiarity of the system is that the roads, as they may be termed, on which the skips of earth travel are not of wire rope, as usual, but of round bar iron. Of these there are two lengths, 5 feet 9 inches apart—one having a diameter of 1¼ inch for the loaded skips, and the other having a diameter of 1.02 inch for the empty ones. They are welded up into lengths of about 55 yards, and these lengths are afterwards united by steel couplings, having a diameter as little differing from their own as possible These "roads" are anchored down to the ground at the loading end, but at the other end terminate in chains which pass over pulleys and support hanging weights, sufficient to bring a strain of 4½ tons on the stronger "road," and 3½ tons on the weaker. There are forty intermediate supports of timber (some of them above 100 yards apart), carrying at the top small grooved pulleys on which the rope actually rests. The skips are made entirely of wrought iron, much like a dredger bucket. They weigh about 290 lbs , and carry 8½ cubic feet, the total weight transported being thus about 1,000 lbs. They are hung on gudgeons, and empty themselves by turning over as soon as a single fastening is loosened. There are special mechanical arrangements for connecting the hanging skip with a small frame above, carrying two wheels which travel on the fixed "road." The whole is connected with a running endless wire rope which communicates the motion. It is ⅝ inch in diameter, is driven by a portable engine at the loading end, and is kept taut by passing round a counterweighted sheave of 5 feet 9 inches diameter, capable of traveling upon cast iron guides. The speed of the running rope is in general about 4 feet per second. Small steel couplings are fixed on it at intervals of 130 feet, and to these the successive skips are attached—the mechanism both for attaching and disengaging being self-acting. At the discharging end there is an apparatus of iron rails for transferring the skips from one road to the other; by making this portable and providing it with supports it is further utilized for shifting the ends of the tramway and the point of loading so as to follow the progress of the excavation. With the speed of rope and distance between skips given above the quantity of earth conveyed is about 40 cubic yards per hour. The changing of the skips at the loading point (and similarly at the discharging point) is managed by a single workman. Each empty skip, as it comes upon the return road, detaches itself just before it reaches the workman, who receives and pushes it on to one of the lines of rails leading to the face of the excavation. He has already put in position a full skip which has just arrived by the other line of rails; and the same coupling which has conveyed the empty skip picks up the full skip in passing, attaches it to the rope and takes it away along the tramway. Thus the whole number of men employed on the tramway, as apart from the excavation, is only six at the most. The engine employed is of 8 HP., but 2 to 3 HP. is found to be all that is required for the work. The laying down of the tramway was completed, though in the depth of winter, in less than ten weeks; and Herr Bleichert, the inventor, states that he has tramways at work on this system which are 2,400 yards long, with gradients of 1 in 3½, and conveying earth to the extent of 500 cubic yard per day.—*Deutsche Bauzeitung.*

SWISS TRIANGULATION.—The operations in the neighborhood of Aarlberg, under the direction of the Spanish General Ibanez, for measuring the base of the network of Swiss triangulation, have been very satisfactory. The base has been measured twice. The result of the first measurement was 2400.087 meters (1.502 miles); the second operation, which was conducted entirely independently of the first, for the purpose of verification, gave 2400.085 meters. The difference between the two measurements was only two millimeters, or less than one ten-thousandth of one per cent. The place chosen for the base is on the route of Sisselen, where a perfectly straight and nearly horizontal line may be drawn for a distance of about three kilometers (1.864 miles). Similar measurements are to be made in the valleys of Tessin and of the Rhine.

THE SUEZ CANAL TONNAGE—Of a total of 3,446,431 tons gross of shipping that passed through the Suez Canal in 1880, the share of England was 3,446,431 tons, or more than three-fourths of the whole. The increase, compared with 1879, was from 2,508 524 English, and a total of 3,236,942 tons. Next to England the most marked augmentation was that of the Russian flag, which rose from 8799 tons to 45,899 tons. French tonnage remained almost stationary, having only advanced from 262,017 tons to 271,598. German rose from 21,548 to 52,551; Austrian, from 51,400 to 103,030; Spanish, from 64,468 to 84,519; Italian, from 94,162 to 104,567; and Dutch from 159,024 to 174,485. All the principal countries of Europe shared in the improved trade with the East in 1880.

IRON AND STEEL NOTES.

A GREAT CRUCIBLE STEEL CASTING.—During the past week Messrs. Jessop & Sons, Brightside Steelworks, Sheffield, cast the largest crucible steel casting that has yet been produced. It is a spur ring 28 feet in diameter, machine moulded, and cast whole. To cast it 270 pots were used, each pot holding 80 pounds weight of molten steel. When the

steel had been poured into the three large ladles, the plugs were removed, and it ran into the mould, the weight when cast being about 10 tons. In its finished state the weight will be about 8½ tons. It is, without doubt, by far the largest crucible cast steel casting of its kind that has ever been produced. Messrs. Jessop & Sons anticipate that this will be the beginning of an important trade with Lancashire millowners, as they discover how much more durable steel wheels are than the cast-iron wheels at present in general use. The firm have previously cast wheels 13 feet and 14 feet in diameter, but to 28 feet was a great leap. Now, however, they are prepared to undertake castings up to 34 feet, having gone to very great expense in laying themselves out for this class of work. The operation of casting occupied eight and a-half minutes.

BESSEMER STEEL IN THE UNITED STATES.—The total production of Bessemer steel during last year, at the eleven works in the United States, during the year 1880, indicates an unusual amount of activity. The amount of steel manufactured in 1880 was 1,203,173 tons (2000 lbs.) showing an increase of 30 per cent. as compared with 1879; of 64 per cent. compared with 1878, and 115 per cent. compared with 1877. The annexed table shows the annual production of Bessemer steel in the United States since 1872:

Year.	Tons.	Year.	Tons.	Year.	Tons.
1872	120,108	1875	375,517	1878	732,226
1873	170,652	1876	525,996	1879	928,972
1874	191,933	1877	560,587	1880	1,203,173

During the year, the eleven companies produced a total of 917,592 tons (2000 lbs) of steel rails, and the development of this manufacture will be seen from the following table, which corresponds to the one above:

Year.	Tons.	Year.	Tons.	Year.	Tons.
1872	94,070	1875	290,863	1878	550,398
1873	129,015	1876	412,461	1879	683,964
1874	144,944	1877	432,169	1880	917,592

Startling as this growth of production is, it will undoubtedly be surpassed by the output of the present year, for not only does the demand continue in the United States, but several of the more important works materially enlarged their capacities last year; the Vulcan Steel Works at St. Louis got into working order only in March last, so that it contributed to the total only during nine months; the Pittsburgh Bessemer Steel Company, with a capacity of 60,000 tons of ingots a year, are just on the point of commencing work; and the Colorado Coal and Iron Company will be in operatowards the autumn.

AT a meeting of the Chemical Society on the 20th ult., a paper was read on a "New Theory of the Conversion of Bar Iron into Steel by the Cementation Process." by Mr. R. Sydney Marsden, D. Sc. The author first referred to a former paper of his in the "Proceedings" of the Royal Society of Edinburgh, Vol. X., p. 712, in which he had shown that when amorphous carbon in an impalpable powder is kept in contact with porcelain at a temperature considerable above redness, but not sufficient for the latter to become fused, the carbon will, if left for a number of hours, diffuse into the porcelain and ultimately permeate it throughout. He considers the conversion of bar iron into steel by the cementation process as analogous to this—that is to say, the result of diffusion of carbon in an impalpable powder into the bars of iron whilst they are in an expanded and softened state. Silicon also appears to be present in the amorphous condition, and to diffuse through iron in a similar manner.

RAILWAY NOTES.

MASSACHUSETTS RAILROADS.—From the Report of the Railroad Commissioners we learn that the aggregate length of the railroads belonging to corporations making returns to the Board is 2667.35 miles, of which 651.48 miles are provided with double track. The total length of track, counting sidings and branches, is 4257.78 miles. The increase during the year being 107.79 miles.

The average cost of standard gauge roads is returned at $57,057.80 per mile, and the cost of equipment averages $6613.90 per mile, making total cost per mile an average of $63,671.70; but this cost varies from $24,353 25 to $92,068.30. The average cost of the equipped narrow gauge roads is $31,293 30.

The gross income for the year, of the sixty-four corporations, was $35,140,374.77, and the net income $11,191,815.53, a gain of 10 per cent. over last year.

The average earning per mile of road was $11,377.90. For the eight roads terminating in Boston the earning per mile averaged $12,079.70.

DURING the early part of last year the total length of the Prussian railway system was 6197 kilometers, including 798 which had been added in the course of the year. By the end of the year the length had been increased to 6299 kilometers. The cost of the whole system had been 1,493,305,418 marks—£74,665,270—or at the rate of 244,312 marks per kilometer. The receipts for the year were 163,877,969 marks as against 155,881,124 in the previous year. There was, however, a diminution of receipts per kilometer from 29,582 marks to 26,580, or at the rate of 9 per cent.; though, on the other hand, the expenditure also decreased from 18,042 to 16,326 marks per kilometer, or 9½ per cent. The general result shows an excess of receipts over expenditure of 61,826,748 marks, against 57,990,555 in the year before.

IN accordance with the provisions of the Berlin Treaty, the Railway Commission appointed to deal with the new lines of the southeast of Europe will assemble in Vienna for the despatch of business in the second half

of February. The Bosnia Valley Railway Bill has passed the Upper Chamber, Austria. This measure is very important, as it will bring Austria into direct communication with the port of Salonica Herr von Schmerling remarked that he looked upon the Austrian occupation of Bosnia not as a temporary but as a permanent thing. This railway, he thought, would help to attach the natives of the province to Austrian rule by identifying their interests with those of the empire.

RAILWAYS AND CANALS IN CANADA.—The report of the Minister of Railways and Canals of Canada for the year ending June 30, 1880, has been presented to the Dominion Parliament. It is stated that on the section of the Canadian Pacific Railway from Thunder Bay to Rat Portage, 294 miles, the railways for 171 miles are laid, and from Kiewatin (Rat Portage) to Selkirk on the Red River, 112 miles, the railways are down the whole way. For a distance of 16 miles beyond Red River, up to Victoria junction, Selkirk, no line has been constructed, but communication is open to Winnipeg by means of the Pembina branch on the east side of the river, and a temporary bridge over the river connects the line with the Winnipeg branch. Westwards the railways are laid to within a few miles of the western boundary of Manitoba, and the line is in operation to Portage-la-Prarie, 70 miles west of Winnipeg. The Canadian Pacific Railway therefore has at the present time 232 miles in work as follows: Emerson to Winnipeg, 63 miles; St. Boniface to Selkirk, 23 miles; Selkirk to Cross Lake (east), 76 miles; Selkirk to Portage-la-Prarie, 70 miles. In British Columbia 127 miles are under contract, and are being steadily prosecuted. The expenditure in connection with the Pacific Railway during the last fiscal year was $4,444,572. The enlargement of the canal system is being rapidly proceeded with, and during last year the expenditure on that account was $2,125,455. The new canals when complete will have a uniform system of locks, 270 feet long by 45 feet wide, with a depth of 14 feet, and will enable steamers of 1500 tons to pass from Lake Superior to Montreal and the sea. The total length of inland navigation from Lake Superior to the Straits of Belle Isle is 2384 miles, and the completion of the new work cannot fail, it is considered, to have an important effect upon the transport of grain from the Western States of America and Western Canada to the United Kingdom.

FRENCH RAILWAYS.—The Director-General of Railways in France has recently published the results of working the main French lines during the first nine months of 1879 and 1880, which returns show a marked improvement between January and September, 1880. The total gross receipts during these nine months amounted to £30,292,000, or nearly 3½ millions more than during the corresponding period of 1879. During the year ending September last, 591 miles of new lines had been opened for traffic. The gross receipts of £30,292,000 were divided among the old and new *reseaux*, the railways belonging to the State and those of private companies as follows: The old *reseau*, 6244 miles in length, earned £22,347,000, or more than 70 per cent. of the total. The new *reseau* of 6253 miles earned £6,798,000. The State railways, 1179 miles long, earned £537,600, and the various private lines, 517 miles in length, have earned the remainder of the total. These results show a large increase over the corresponding period of 1879; thus the old *reseau* shows an increase of £2,486,000; the new *reseau* a rise of £791,000, and the State Railways a rise of nearly £50,000. The line from Rhone and Mont Cenis Railway classified in the Paris, Lyons, and Mediterranean system, earned £167,360 during the first nine months of 1880, or nearly £18,000 more than in the similar period of 1879; the former figure represents an earning of over £2300 per mile of line.

ORDNANCE AND NAVAL.

AN HYDRAULIC SHIP.—Another hydraulic ship has lately been launched in Germany, but as usual with vessels of the same description, she has failed to attain the speed expected. As the readers of *Iron* are aware, in these hydraulic ships water taken in through the bottom is expelled at both sides in the line of the keel, and the reaction of the fluid issuing at high speed drives the hull ahead if the water is allowed to escape aft, or astern if it escapes towards the bow. The notion is no new one, for as far back as 1661 an inventor received a patent for propelling vessels by expelling water from their sterns. In 1730 another patent was secured for doing nearly the same thing, and since that time nearly fifty persons have taken out patents for hydraulic vessels. About ten years ago a vessel called the "Waterwitch," of 1279 tons displacement, was built for our navy, to test a proposed system of hydraulic propulsion, but, though the ship is only of light draught, has good lines, and is fitted with engines of 775 horse power, she has never exceeded, when at sea, a speed of from five to six knots, and has never been trusted out of sight of land. The hydraulic ship lately built in Germany is 110 feet long, 17 feet wide, with a draught of 15½ feet of water, and was expected to attain a speed of ten knots an hour, but on her trial trip she could barely accomplish nine. She proved, however, to be extremely handy, running close up to a sailing vessel which crossed her course, and then, stopping her way in a moment, turned immediately to starboard.

THE AUSTRIAN IRONCLAD "TEGETHOFF."—A trial of a new steam fire engine, designed for the Imperial Austro-Hungarian warship Tegethoff took place on Tuesday at Messrs. Merryweather and Sons' fire-engine works at Greenwich. The engine is fitted with a quick-raising steam boiler of the Merryweather and Field improved type. The pump is provided with valves having large clear openings through which seaweed, shavings,

straw, and other foreign matter may pass without fear of injury or stoppage to the machinery. At the trial steam was raised within eight minutes from cold water, and the engine pumped 360 gallons per minute through a 1¼-inch jet to a height of 160 feet. Following this test two, three, and four jets were thrown by the engine simultaneously, and as an experiment, to show its capabilities for pumping powers the machine discharged an immense body of water through two 2½-inch hoses. This last experiment was perhaps the most important to the visitors present, as the primary work of the engine (although a fire engine to all intents and purposes) is to pump out water in case of a mishap, damage by torpedo or shot. Being complete in itself, such an engine by means of suction pipes can draw water from any damaged compartment, and keep it down until the necessary repairs are made.

THE NEW FORTY-THREE TON BREECH-LOADING GUN.—The heavy breech-loading gun now in course of its departmental trials in the Royal Gun Factories naturally excites much interest, being the first breech loader in our service larger than the old Armstrong 7-inch guns, which fired shot weighing 110 pounds, with considerable difficulty, and were eventually supplied with shells under 100 pounds in weight. The new gun weighs about 43 tons. Its caliber is 12 inches, and its length of bore about 26 calibers—the whole gun being about 29 feet long. The method of closing the breech is the same as that adopted in the new field breech-loading guns, except that lever and screw power is applied to the closing and locking of the breech. In general appearance, proportions, and character, the gun closely resembles the 9.2 inch breech-loading Woolwich gun depicted in *The Engineer* of June 25, 1880. The new armor-clad vessels Colossus and Majestic will each, we believe, carry four of this description of gun, mounted in two turrets, arranged similarly to those of the Inflexible, which vessel they will closely resemble. They are to be 5 feet longer and 7 feet narrower, and are to be made of steel, with compound armor 16 inches thick. For a full description the reader is referred to King's "War Ships," new edition. Up to the present time the gun in question has not been the subject of any public trial, being, as we have said, in the course of undergoing its early proofs in the hands of Colonel Maitland, the Superintendent of the Gun Factories. In this stage it is impossible to give full detailed information. We think, however, that we can supply what is sufficiently near for all practical purposes. The gun was fired on Thursday, January 6th, with about 280 pounds of prismatic powder, which, with a projectile weighing 703 pounds, gave an initial velocity of about 1830 feet per second. The charge is being gradually increased—the pressure in the bore being as yet only about 16 tons on the square inch. Taking it in its present condition, however, the gun is a very powerful weapon, having about 16,330 foot-tons stored-up work, or 435 foot-tons per inch circumference, which means a penetration of from 22 inches to 23 inches—on the service system of calculation, nearly 25 inches. This is the class of gun we have before commended as likely to be useful in future times. It is capable of piercing wrought iron of the thickness of the armor employed on the Duilio. As a matter of fact, the plates of that ship are steel, so that they could not be penetrated, but must be destroyed by racking. For its size, the racking power of the 43-ton gun is very great. The 35 ton was of the same caliber, it had 8203 foot-tons stored-up work, a penetrating figure of 219-06 foot-tons, and about 16-inch penetration. Thus, for 8 tons increased weight the new gun nearly doubles the quantity of stored-up work and the penetrating figure. We must not blink the fact that 29 feet is a length of gun involving special allowance in the turret and deck. The maximum width of both the Colossus and Majestic is 68 feet. The power of this piece then is very great, and we have not yet reached the maximum development. On the most recent trials a velocity of 1930 feet was obtained with 300 pounds of powder, and a pressure of 19 tons on the square inch.—*Engineer.*

BOOK NOTICES.

PUBLICATIONS RECEIVED.

ILLUSTRATED CATALOGUE OF THE BALDWIN LOCOMOTIVE WORKS.

OFFICIAL GAZETTE OF THE UNITED STATES PATENT OFFICE. Vol. 19, No. 7.

TWELFTH ANNUAL REPORT OF THE BOARD OF RAILROAD COMMISSIONERS OF MASSACHUSETTS.

MONTHLY WEATHER REVIEW FOR JANUARY, 1881.

THE WORKSHOP. No. 3. E. Steiger & Co.

THROUGH the kindness of Mr. James Forrest, Secretary of the Institution of Civil Engineers, we have received the following published papers of the Institution.

"Address of James Abernethy, Esq., F. R. S. E."

"Machinery for Steel Making," by Benj. Walker, M. I. C. E.

"The Monte Penna Wire Ropeway," by William P. Churchward, A. I. C. E.

"Cribwork in Canada," by Henry Taylor Bovey, M. A.

"Different Modes of Erecting Iron Bridges," by Theophilus Seyrig, M. I. C. E. Translated by Alfred Bache, A. I. C. E.

LOCOMOTIVE ENGINEERING AND THE MECHANISM OF RAILWAYS. By Zerah Colburn. For sale by D. Van Nostrand. Price $10.

This is not a new book, nor is it a new edition of an old one, but attention is called anew to this valuable treatise by the announcement of a material reduction in the price. Two large quartos, describing and illustrating in

the fullest manner every department of locomotive structure and use, are now offered for ten dollars.

The plates afford working dimensions, and the text fully describes the details.

CAOUTCHOUC; GUTTA PERCHA ET GOMME FACTICE. Paris: Librarie de Roret. For sale by D. Van Nostrand. Price $1.75.

There are two volumes of this little French work, of which the first is devoted to caoutchouc. The methods employed in preparing the raw material, and the machinery for the manufacturing processes are briefly described. The plates are good, although few in number.

NOUVEAU TRAITE DE CHIMIE INDUSTRIELLE. Wagner et L. Gautier. Paris: Librairie F. Savy. For sale by D. Van Nostrand. Price $10.50.

This extensive work is now in two large octavo volumes illustrated with the best style of woodcuts and in the most profuse manner, there being 487 such illustrations in the work.

The first section of the work treats of Metallurgical Processes; the second, of Raw Materials and Products of Industrial Chemistry; the third, of Glass, Pottery, Lime and Plaster; fourth, Vegetable Matters and their Industrial Applications; fifth, of Animal Matter; sixth, Coloring Matters; seventh, Illumination; eighth, Heating, the Materials and Apparatus.

LIFE HISTORY OF OUR PLANET. By Wm. D. Gunning. New York: R. Worthington. Price $1.50.

This is paleontology in a popular, not to say an unscientific form. Facts are gleaned here and there from the zoology of the past, and presented in the style of the popular scientific lecturer.

There is a tendency in all such works to give undue prominence to the astounding facts, and to omit the expression of their logical connection; but it may be urged that by such means the attention of the unlearned is first arrested and directed to the proper sources of information.

The book is fairly illustrated, and will, we have no doubt, be gladly received in those sections of the country where the author has been heard on the lecture platform.

EASY LESSONS IN SANITARY SCIENCE. By Joseph Wilson, M. D. Philadelphia: Presley Blakiston. Price $1.00.

Land Drainage, Drainage of the Farm House and Village, Drainage of Cities, and Plumbing, are the topics treated in an easy colloquial style by this author.

The writer possesses sufficient knowledge, derived from experience, and has produced a valuable, though brief and badly illustrated treatise.

MATERIALS AND CONSTRUCTION. By Francis Campin, C. E. London: Crosby Lockwood Co. For sale by D. Van Nostrand, Price $1.20.

This is a new volume of the well-known Weale series. It is prepared by a well-known author for those readers who desire to become thoroughly acquainted with theories of structures, and the practical application of results in the simplest way, and not as a mathematical exercise.

A convenient and compact volume is the result of this plan, containing as much science as a large proportion of working engineers are accustomed to employ.

THE STEAM ENGINE AND ITS INVENTORS. By Robert L. Galloway. London: Macmillan & Co. Price $3.50.

This is a brief history of the use of steam from 1660 to 1820. The object of the author, as explained in the preface, is to exhibit the successive steps in the development of the cylinder and piston engine, and not to give an account of all the machines in which steam has been employed.

The book is well printed, and the illustrations are excellent, although the latter will look familiar to readers of former sketches on this subject, and the number will seem small compared with the abundance afforded in Thurston's history of the steam engine.

The bibliography of the subject seems to be fully given in the foot notes, a fact that will render the work valuable to students or writers upon subjects relating to the history of engineering progress.

THE MARINE MAMMALS OF THE N. W. COAST OF NORTH AMERICA. By Charles M. Scammon. New York: G. P. Putnam's Sons. Price $10.00.

This is an illustrated quarto, giving, in addition to the habits of the whales, seals and Dolphins, an account of the methods of the American whale fishery.

The book will prove valuable to the naturalist seeking for full information on these subjects, and will also prove acceptable to any lover of natural history by reason of the fullness of the descriptive text and illustrations.

SEWING MACHINERY. By J. W. Urquhart, C. E. London; Crosby Lockwood & Co. For sale by D. Van Nostrand. Price 80 cents.

This is a late addition to the Weale series, devoted exclusively to an historical and descriptive account of the sewing machine. It will doubtless prove interesting to many, and valuable to a few who need in one book the information heretofore scattered through the technical journals of several years.

MISCELLANEOUS.

WATERPROOF paper is made by a new German method as follows: To a weak solution of ordinary glue add a little acetic acid; then make another solution by dissolving a small quantity of bicromate of potash in distilled water. These two liquids should be we

mixed together, and the sheets of paper which have to be made waterproof drawn through the mixture, and suspended from suitable lines to dry. The proportions are not given, but 5 per cent of acetic acid and 7 per cent. of a saturated solution of bicromate of potash will answer.

THE manufacture of bicycles and velocipedes has become one of the staple industries of the Midlands, and lately the railway charges for the carriage of these products have been considerably increased, in some cases up to 100 per cent. The manufacturers of Wolverhampton and Coventry have consequently interviewed the managers of the London and North-Western, the Great Western, and the Midland Railways, seeking a return to the rates that prevailed before 1881. The result has been that with this month the recently advanced rates have been lowered 50 per cent. on bicycles, and something like 75 per cent. on tricycles. The manufacturers in the Wolverhampton district have formed themselves into an association.

DIURNAL VARIATIONS OF THE BAROMETER.—G. H. Simonds has computed the barometric coefficients of daily variations from observations made at thirty stations scattered over the globe. The average duration at each station was something over ten years. Under the tropics the effect of latitude is scarcely perceptible, but between the latitudes of 20° and 60° the value of the quadrantal component decreases .001 inch per degree of latitude. The calculations and the discussions which have resulted from them confirm Sir John Herschel's views, in regard to the universality of the phenomenon, and Chase's explanation of the cause.

CIRCULATION OF AIR IN THE ST. GOTTHARD TUNNEL.—M. Stapf has been giving careful attention to the variations in the air currents between the two openings at Göschenen and Airolo. He finds two principal causes to be operative in these changes: First, the southern opening is 30 meters (32.809 yards) higher than the northern, which represents a pressure equivalent to that of a column of air 36 meters (39.371 yards) high at the subterranean temperature; second, the difference of barometric pressures upon the two declivities of the mountain. If the external temperature was always lower than the internal, if the barometric temperature was the same at each side, and if there were no modifications of velocity due to the heating and expansion of the air or to the friction against the walls, the draft would always be southward. Meteorological observations are regularly taken, both at Airolo and Göschenen, to determine the elements which are required in order to know, monthly or annually, the number of days for which a given direction of current or an absolute calm may be expected in the interior of the tunnel.

THE new law upon trade marks now in force in Switzerland embraces three points of procedure, viz.: deposit, registry, and renewal. The act of deposit—made at the Federal Bureau at Berne—must be accompanied by a legal attestation as to the place of manufacture; the trade mark in triplicate, with exact description of the article; a block of the trade mark, of a specified size and thickness; and lastly, a sum of 20f. for each separate trade mark. The deposit can be made by a third party by procuration. The registration must contain information as to the number of the trade mark; the data of deposit and publication; the name, profession, and address of the proprietor; the description of the article and of any modifications which might have been made since the day of deposit. The description is given in the language of the depositor, provided it is in one of the three national languages; if otherwise, in French. Renewal is accompanied by the same formalities as deposit. The law on trade marks does not deal with, or affect in any way, the law of patents, which which will form a subject of debate in the renewal of commercial treaties between Switzerland and the neighboring States. *Apropos* of this, says a contemporary, it may be mentioned that an international conference upon the law of patents has just been sitting in Paris, in which M. Kern, the Swiss Minister to France, has taken part.

COLOR RELATIONS OF METALS.—In a paper on the color relations of copper, nickel, cobalt, iron, manganese, and chromium, lately read before the Chemical Society, Mr. T. Bayley records some remarkable relations between solutions of these metals. It appears that iron, cobalt, and copper form a natural color group, for if solutions of their sulphates are mixed together in the proportions of 20 parts of copper, 7 of iron, and 6 of cobalt, the resulting liquid is free from color, but is grey and partially opaque. It follows from this that a mixture of any two of these elements is complementary to the third, if the above portions are maintained. Thus a solution of cobalt (pink) is complementary to a mixture of iron and copper (bluish green); a solution of iron (yellow) to a mixture of copper and cobalt (violet); and a solution of copper (blue) to a mixture of iron and cobalt (red). But, as Mr. Bayley shows, a solution of copper is exactly complementary to the red reflection from copper, and a polished plate of this metal viewed through a solution of copper salt of a certain thickness is silver white. As a further consequence, it follows that a mixture of iron (7 parts) and cobalt (6 parts) is identical in color with a plate of copper. The resemblance is so striking that a silver or platinum vessel covered to the proper depth with such a solution is indistinguishable from copper.

There is a curious fact regarding nickel also worthy of attention. This metal forms solutions, which can be exactly simulated by a mixture of iron and copper solutions; but this mixture contains more iron than that which is complementary to cobalt. Nickel solutions are almost complementary to cobalt solutions, but they transmit an excess of yellow light. Now the atomic weight of nickel is very nearly the mean of the atomic weight of

iron and copper, but it is a little lower, that is, nearer to iron. There is thus a perfect analogy between the atomic weights and the color properties in this case. This analogy is even more general, for Mr. Bayley states that in the case of iron, cobalt, and copper, the mean wave length of the light absorbed is proportional to the atomic weight. The specific chromatic power of the metals varies, being least for copper. The specific chromatic power increases with the affinity of the metal for oxygen. Chromium forms three kinds of salts. Pink salts, identical in color with the cobalt salts; blue salts, identical in color with copper salts; and green salts, complementary to the red salts.

Manganese, in like manner, forms more than one kind of salt. The red salts of manganese are identical in color with the cobalt salts, and with the red chromium salts. The salts of chromium and manganese, according to the author, are with difficulty attainable in a state of chromatic purity. He thinks these properties of the metals lead up to some very interesting considerations,

JAPANESE LEATHER PAPER.—Among the numerous forms in which Japanese art has been so popularly represented in this country of late years there are few proving so practically useful as the subject of this notice. Introduced at a time when startling innovations were being made in matters of taste, it readily lent itself to the efforts of designers who were gifted with fresh and original ideas. With no associations wedding it to any particular style, and with a character which neither asserts nor obtrudes itself, it harmonizes with almost every style of decoration. The paper is rich and generous in color and charming in design. Some have wonderful bits of detail, amidst a mass of suggested form and foliage; others show more than a suspicion of European influence. One of the latest now before us has all the familiar features of the best old Spanish leather, but accentuated in a manner both novel and characteristic. But whether drawn from their own resources or elaborated on accepted forms, they are peculiarly adapted for the purposes for which we recommend them, viz., interior decoration, such as dados, entrance-halls, staircases, &c. But from their variety, both in design and color, they can also be used with fine effect wherever decoration is introduced. Apart from its value as a decorative agent, Japanese leather paper possesses in a large degree the desirable quality of durability, and in this respect thoroughly deserves it name, being practically indestructible when once fixed on a wall. The origin of this interesting material in its present form may be regarded as a legitimate outcome of one of our International Exhibitions. Before 1872 specimens of the material were only to be found in the hands of dealers or collectors. But in 1872 two or three pieces of the then orthodox size, that is, 18 inches by 12 inches, in no way striking in pattern, were exhibited in a case at South Kensington, where they were regarded only as an object of curiosity. Mr. Robert Christie, recognizing its value as a material for the purposes for which it is now so largely used, and having at that time business connections with Japan, at once endeavored to find out the makers there, with the view of getting it produced in a form more useful for European requirements. His efforts were so far successful that he was enabled to exhibit several rolls of different patterns, measuring 3 feet wide by 36 feet long at the Exhibition in 1874, and for which he obtained a medal. Since then Mr. Christie has been in constant correspondence with the makers in Japan, and, having had practically the monopoly of its manufacture up to the present time, has used his influence in impressing on them the desirability of adhering as near as possible to their own old colors and designs. Although the manufacture of this material has been largely developed, its reproduction has in no manner deteriorated, unlike other native productions both of India and Japan, where trade competition and indiscriminate orders have had such a baneful influence on their art work; the only deviation from this rule being as regards size, in order to do away with the patchy and unsatisfactory appearance which was found to be unavoidable where the original small pieces were used, particularly in cases where the pattern ought to, but did not intersect. We are glad to hear from Mr. Christie that its sale at the present time is largely increased both in this country and America, and it is now regarded by an intelligent class of decorators as one of the most promising staples of trade with Japan.

THE Phosphor-Bronze Company held its seventh annual general meeting on the 1st inst. The directors declared a dividend of 10 per cent. per annum, less the interim dividend—2½ per cent.—already paid. The directors have devoted, they say, considerable attention to several novel applications of phosphor-bronze—amongst others, to its adaptability in the form of sheet, angle bars, and rivets, to the construction of steam launches and torpedo boats; and having thought it desirable to make a practicable trial in this direction, they have ordered a small steam launch to be built entirely of phosphor-bronze, to test its greater durability by reason of its resistance to the oxidizing effect of sea water. Should the venture prove a success, there is every likelihood of a considerable business developing in this branch.

THE agreement amongst the coalowners of Germany has been revived. Where the limit of 45,000 tons in the output has not been exceeded, the same production is allowed for 1881. New members of the ring must undertake to reduce their output by 5 per cent., and to agree to a fine of 1f. 25c. for every ton above 45,000. The fine is, however, in no case to exceed 125,000f.